85. *On Geometrical Classification of Fibers in Pencils of Curves of Genus Two*

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(Comm. by Kôsaku YOSIDA, M. J. A., June 2, 1972)

All possible numerical types of fibres in pencils of curves of genus two have been classified by Ogg ([9])**) and Iitaka ([5]) independently, and recently Winters has shown that all of them arise actually as a corollary of his more general existence theorem ([12]).

This article contains their geometrical classification which is essentially different from the numerical one. By using our method it can be also shown that all possible types arise. Our method should be generalized for pencils of curves with arbitrary genus. The complete classification and explicit construction of fibres will be shown in our forthcoming papers ([7], [8]).

1. Construction of geometrical invariants and characterization of fibres by them.

(1) Let $\pi: X \to D$ be a pencil of (complete) curves of genus two over a disc $D=\{t \in \boldsymbol{C}; |t|<\varepsilon\}$. Further, we assume the following:

i) X is a non-singular (complex analytic) surface free from exceptional curves of the first kind;

ii) π is smooth over a punctured disc $D'=D-\{0\}$. Thus for every $t \in D'$ the fibre $X_t=\pi^{-1}(t)$ is a complete non-singular curve of genus two.

In this article we consider only such pencils.

(2) (See [10] for detailed discussion in this paragraph.) For every $t \in D'$ denote by J_t the jacobian variety of X_t. Then the J_t's form a holomorphic family $\boldsymbol{J}$ of abelian varieties of dimension two over D'. Moreover $\boldsymbol{J}$ is a polarized bundle in the sense of [10]. Therefore we can construct a canonical multivalued holomorphic map $T_\pi: D' \to \mathfrak{S}_2$ where $\mathfrak{S}_2$ is the Siegel upper-half plane of degree two.

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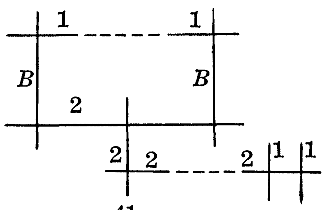
*) Partially supported by the Sakkokai Foundation.

**) In Ogg's table the following types are missing.



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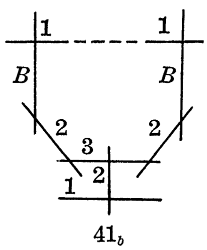

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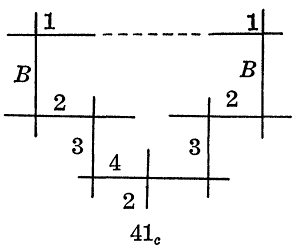

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