84. *Rigidity for Elliptic Isometric Imbeddings*

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Introduction. We say that an isometric imbedding f of a Riemannian manifold M to the Euclidean space $\boldsymbol{R}^m$ is elliptic if the imbedding f is generic in a suitable sense and if the differential operator L associated with f is elliptic. We then establish a rigidity theorem (Theorem 2) for elliptic isometric imbeddings of compact Riemannian manifolds to $\boldsymbol{R}^m$. Furthermore we apply this result to the rigidity problem associated with the canonical isometric imbedding f of a compact hermitian symmetric space $M=G/H$ to the Euclidean space $\boldsymbol{R}^m$, where $m=\dim G$ (see Theorems 3 and 4). Theorem 4 partially generalizes the classical theorem of Cohn-Vossen.

1. Throughout the present paper we shall always assume the differentiability of class C^∞.

Let M be an n-dimensional manifold. T denotes the tangent bundle of M and T^* its dual. S^2T^* denotes the vector bundle of symmetric tensors of type $\binom{0}{2}$ on M. Given a vector bundle E on M, $\Gamma(E)$ denotes the space of cross-sections of E. Let $\boldsymbol{R}^m$ be the m-dimensional Euclidean space. $\langle\ ,\ \rangle$ denotes the inner product on $\boldsymbol{R}^m$ as a Euclidean vector space.

Let $\Gamma(M, m)$ be the vector space of all the maps $\boldsymbol{u}$ of M to $\boldsymbol{R}^m$ and $\mathfrak{E}$ the subset of $\Gamma(M, m)$ consisting of all the imbeddings f of M to $\boldsymbol{R}^m$. We assume $\mathfrak{E}\neq\phi$. For any $f\in\mathfrak{E}$, we denote by $\Phi(f)$ the Riemannian metric on M induced by the imbedding f:

$$\Phi(f)=\langle df, df\rangle=\sum_k (df_k)^2,$$

where $f=(f_1, \cdots, f_m)$. Then the assignment $f\to\Phi(f)$ gives a map Φ of the set $\mathfrak{E}$ to the set $\mathfrak{M}$ of all the Riemannian metrics on M. For any $f\in\mathfrak{E}$, we define a differential operator Φ_{*f} of $\Gamma(M, m)$ to $\Gamma(S^2T^*)$, the differential of the map Φ at f, by

$$\Phi_{*f}(\boldsymbol{u})=2\langle df, d\boldsymbol{u}\rangle \quad (\boldsymbol{u}\in\Gamma(M, m)).$$

2. Let f be an imbedding of M to $\boldsymbol{R}^m$. We put $\nu=\Phi(f)$. Let N be the normal vector bundle on M associated with the imbedding f; the fibre N_p over a point p of M may be identified with a subspace of the Euclidean vector space $\boldsymbol{R}^m$. It is well known that, for any vectors $X, Y\in T_p$, the derivative $\nabla_X\nabla_Y f$ is in the subspace N_p of $\boldsymbol{R}^m$, where ∇ is the covariant differentiation associated with the Riemannian metric ν.