102. *A Note on Infinitesimal Generators and Potential Operators of Contraction Semigroups*

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1. Introduction. Hirsch [3] proves that an operator V in a Banach space is the "cogenerator" $\lim_{\lambda\to 0} J_\lambda$ of a pseudo-resolvent J_λ satisfying $\lim_{\lambda\to 0}\lambda J_\lambda=0$ if and only if $-V$ is the "generator" $\lim_{\lambda\to\infty}\lambda(\lambda J'_\lambda-1)$ of a pseudo-resolvent J'_λ satisfying $\lim_{\lambda\to\infty}\lambda J'_\lambda-1=0$. He notices a dual relation between J_λ and J'_λ. For semigroups, such a duality is not obtained between infinitesimal generators (i.g.) and potential operators (p.o.). However, the situation is rather simple in the case of contraction semigroups in Hilbert spaces, which is implicit in Hirsch [2]. In this note we give the result more explicitly, and also give a connection with Phillips' characterization of i.g. Further we consider contraction semigroups in Banach spaces.

2. Hilbert space. Let $\mathfrak{X}$ be a Hilbert space (real or complex). We mean by a contraction semigroup a strongly continuous semigroup of linear contraction operators on $\mathfrak{X}$. A contraction semigroup T_t with resolvent $J_\lambda(\lambda>0)$ and i.g. A is said to admit a p.o. if the set of f such that $J_\lambda f$ strongly converges as $\lambda\to 0$ is dense in $\mathfrak{X}$. If this is the case, the operator V defined by the limit is called the p.o. and satisfies $V=-A^{-1}$ (Yosida [11]). An operator A is called dissipative if $\mathrm{Re}\,(f, Af)\leqq 0$ for all $f\in\mathfrak{D}(A)$, and maximal dissipative if in addition no proper extension of it is dissipative. The Cayley transform C of A is defined by $C=(1+A)(1-A)^{-1}$ (Phillips [5]). $\mathfrak{D}, \mathfrak{R}$, and $\mathfrak{N}$ denote domain, range, and null space of an operator, respectively.

Theorem 1. *Let A be a linear operator in $\mathfrak{X}$. Then the following six conditions are equivalent:*

(i) *A is the i.g. of a contraction semigroup admitting a p.o.*

(ii) *$-A$ is the p.o. of a contraction semigroup.*

(iii) *A is maximal dissipative with $\mathfrak{D}(A)$ and $\mathfrak{R}(A)$ both dense.*

(iv) *A is dissipative, $\mathfrak{R}(1-A)=\mathfrak{X}$, and $\mathfrak{D}(A)$ and $\mathfrak{R}(A)$ are both dense.*

(v) *$(1-A)^{-1}$ is defined on $\mathfrak{X}$ and the Cayley transform C of A is a contraction operator with $\mathfrak{N}(C+1)=\mathfrak{N}(C-1)=0$.*

(vi) *There is a linear contraction operator C with $\mathfrak{D}(C)=\mathfrak{X}$ and $\mathfrak{N}(C+1)=\mathfrak{N}(C-1)=0$ such that $A=(C-1)(C+1)^{-1}$.*

Suppose that the above conditions are met, and let $T_t^{(1)}$ and $T_t^{(2)}$ be