134. On a Fine Capacity Related to a Symmetric Markov Process

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§1. Introduction and main results.

Let X be a locally compact Hausdorff space with a countable base and m be a positive Radon measure on X. Let $M=(X_t, P_x, \zeta)$ be an m-symmetric standard process on X. Throughout this paper we make the following assumption:

(A) The measure m is a reference measure for M.

By virtue of (A) and the symmetry of M, it follows from Theorem 1.4 in [1; Chap. 6] that M is self-dual in the sense of [1; Chap. 6]. Further polarity and semipolarity of a set are equivalent (Proposition 4.10 in [1; Chap. 6]). Hence every fine Borel set is nearly Borel because under (A) every fine Borel set is the union of a Borel set and a semipolar set ([1; Chap. 5]).

The expression "*q.e.*" will mean "except on a polar set". A function u defined q.e. on X is called *q.e. finely continuous* if there exists a nearly Borel polar set B such that u is finely continuous on $X-B$. Denote by $(X, m, \mathcal{F}, \mathcal{E})$ the Dirichlet space generated by the m-symmetric resolvent $\{G_\alpha, \alpha>0\}$ of M in the sense of Fukushima [2; §2]. Our main results are the following.

Theorem 1. *Every function in $\mathcal{F}$ has a q.e. finely continuous modification: for every $u \in \mathcal{F}$, there exists a q.e. finely continuous function u^* such that $u^*=u$ m-a.e.*

Denote by $\mathcal{F}^*$ the set of all q.e. finely continuous modifications of functions of $\mathcal{F}$. For each $\alpha>0$, set $\mathcal{E}_\alpha(u, v)=\mathcal{E}(u, v)+\alpha(u, v)$ for $u, v \in \mathcal{F}$, where (u, v) denotes the inner product in $L^2=L^2(X, m)$.

Theorem 2. *If $\{u_n\}$ is a Cauchy sequence in the Hilbert space $(\mathcal{F}^*, \mathcal{E}_1)$, then there exists a subsequence which converges q.e. on X to a function $u \in \mathcal{F}^*$. Furthermore $\{u_n\}$ converges to u with $\mathcal{E}_1$-norm.*

For a finely open set A, let

$$\mathcal{L}_A=\{u \in \mathcal{F};\, u \geqq 1 \quad m\text{-a.e. on } A\} \tag{1.1}$$

and define

$$\operatorname{cap}(A)=\begin{cases}\inf_{u\in\mathcal{L}_A} \mathcal{E}_1(u, u) & \text{if } \mathcal{L}_A \neq \phi \\ =\infty & \text{if } \mathcal{L}_A=\phi.\end{cases} \tag{1.2}$$

For any subset B of X, define