132. On Characters and Unipotent Elements of Finite Chevelley Groups

By Noriaki KAWANAKA
Department of Mathematics, Osaka University

(Comm. by Kunihiko KODAIRA, M. J. A., Oct. 12, 1972)

The purpose of the present paper is to give some results concerning (complex) characters and unipotent elements of finite Chevelley groups. Main results are proved by two simple lemmas stated in section 1. Throughout the paper G denotes a connected reductive linear algebraic group defined over a finite field k of q elements. For simplicity we also assume that G has a maximal torus T which splits over k. If L is an algebraic subgroup of G defined over k, $L(k)$ denotes the finite group of its k-rational elements. If S is a finite set, $|S|$ denotes the number of its elements. For a finite group H and class functions ϕ_1 and ϕ_2 on H, the inner product $(\phi_1, \phi_2)_H$ is defined by

$$(\phi_1, \phi_2)_H = \sum_{x \in H} \phi_1(x)\overline{\phi_2(x)}/|H|.$$

If K is a subgroup of H and θ is a class function on K, $i[\theta; K \to H]$ (or $i[\theta]$) denotes the class function on H induced by θ.

1. Let W be the Weyl group of G relative to T and B a fixed Borel k-subgroup of G containing T. B determines a set Φ_+ of positive roots and a set Δ of simple roots in the system Φ of roots of G relative to T. For each subset δ of Δ, let P_δ be the parabolic k-subgroup corresponding to δ and G_δ, U_δ its Levi k-subgroup and unipotent radical (see § 3 of the paper of A. Borel and J. Tits in Publ. de Math. I. H. E. S. n°27 (1965)). G_δ is connected reductive and the root system Φ_δ of G_δ relative to T is spanned by δ. We denote by W_δ the Weyl group of G_δ relative to T.

Lemma 1 (L. Solomon, C. W. Curtis). (a) *Let 1_δ be the 1-character of W_δ and ε the alternating character of W. Then*

$$\varepsilon = \sum_{\delta \subset \Delta} (-1)^{|\delta|}\, i[1_\delta; W_\delta \to W].$$

(b) *Let $P^1_\delta(k)$ be the set of unipotent elements of $P_\delta(k)$ and θ_δ be the class function on $P_\delta(k)$ defined by*

$$\theta_\delta(x) = \begin{cases} 1 & \text{if } x \in P^1_\delta(k), \\ 0 & \text{otherwise.} \end{cases}$$

If we put

(1.1) $$\Theta = \sum_{\delta \subset \Delta} (-1)^{|\delta|}\, i[\theta_\delta; P_\delta(k) \to G(k)],$$

then

$$\Theta(x) = \begin{cases} q^m & \text{if } x = 1, \\ 0 & \text{otherwise,} \end{cases}$$