153. *Finitary Objects and Ultrapowers*

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(Comm. by Kinjirô KUNUGI, M. J. A., Nov. 13, 1972)

Introduction. When we deal with categories of systems with structures, we often feel it desirable to set up a notion that distinguish algebraic structures, such as ordered sets or groups, from infinitistic theories, such as topological spaces. One attempt was made in [4] for concrete categories and studied particularly in connection with ultrapowers. Here the notion of finitary objects defined in [4] for concrete categories, together with one of the theorems concerning them and ultrapowers of objects, is generalized to abstract categories. Only the definitions and the results are given below. The proofs and further details are to be referred for to a paper with the same title which will be published elsewhere, of which this is an abstract.

As for the terminology, we mostly follow Isbell [2] and the terms "*extremal monomorphisms*", "*small complete*", "*left complete*", "*locally small*", "*strict monomorphisms*" etc., are used in his sense without citing the definitions. "*The co-intersection of quotient objects*" is the dual notion of "the (representable) intersection of subobjects" in [2] and "An object *co-generates* another" is the dual statement of "An object generates another" of Grothendieck [1].

§1. Finitary objects. Let C be an abstract category and Ob (C) the collection of all objects in C.

Definition 1. Let $L=\{b_\lambda : X_\lambda \to B \mid \lambda \in \Lambda\}$ be a set of coterminal morphisms in C. L is said to *cover* B if there is no proper extremal monomorphism $\cdot \to B$ that factors all $b_\lambda \in L$. The set L is called *compatible with* $K=\{a_\lambda : X_\lambda \to A \mid \lambda \in \Lambda\}$, if there exists an $f: B \to A$ such that $a_\lambda = f b_\lambda$ for every $\lambda \in \Lambda$. L is called *finitely compatible with* K, if for any finite subset M of Λ, $\{b_\lambda \mid \lambda \in M\}$ is compatible with $\{a_\lambda \mid \lambda \in M\}$. A is called *finitary under* B, if for any sets $L=\{b_\lambda : X_\lambda \to B \mid \lambda \in \Lambda\}$ and $K=\{a_\lambda : X_\lambda \to A \mid \lambda \in \Lambda\}$, L is compatible with K, provided the former covers B and finitely compatible with the latter. A is called *finitary*, if it is finitary under every B in Ob (C).

It can be seen that in the category of groups or of ordered sets, or in more general, in that of models of an algebraic theory, of which all primitive relations are finitary, every object is finitary, as is the intension of Definition 1, while, in the category of Hausdorff spaces, even a finite set, save for the singleton space, is not finitary.