22. *Semi-linear Poisson's Equations*

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§1. Semi-linear Poisson's equations. Let S be a separable, locally compact, non-compact Hausdorff space, and $C_0(S)$ be the completion with respect to the maximum norm of the space of real-valued continuous functions with compact supports defined on S. $C_0(S)$ is thus a Banach lattice.[1] Assume that we are given a *"non-negative" contraction semi-group* $\{T_t\}_{t\geqslant 0}$ *of class* (C_0) *in* $C_0(S)$ (see Phillips [11], Hasegawa [5] and Sato [12]). We shall be concerned with the situation in which

(1) the infinitesimal generator A of $\{T_t\}_{t\geqslant 0}$ admits a densely defined inverse A^{-1}.

That is, we suppose that the *semi-group* $\{T_t\}_{t\geqslant 0}$ *admits a "potential operator" V in the sense of Yosida* [17] (see also Chapter XIII, 9 of Yosida [19]):

$$V=-A^{-1}.$$

Now we introduce a nonlinear operator[2] β_0 in $C_0(S)$ associated with a strictly monotone increasing continuous function $\beta\colon D(\beta)=(a,b)\to R^1$, $-\infty\leq a<0<b\leq+\infty$, such that $\beta(0)=0$, $\lim_{r\downarrow a}\beta(r)=-\infty$ if $a\neq-\infty$, and that $\lim_{r\uparrow b}\beta(r)=+\infty$ if $b\neq+\infty$:

$$(2)\qquad \begin{aligned} &D(\beta_0)=\{u\in C_0(S)\,;\,u(s)\in D(\beta) &&\text{for any } s\in S\},\\ &(\beta_0 u)(s)=\beta(u(s)),\ s\in S, &&\text{for } u\in D(\beta_0). \end{aligned}$$

We consider the "semi-linear Poisson's equation":

$$Au-\beta_0 u=-f,\qquad f\in C_0(S).$$

Our theorem of the existence and uniqueness reads:

Theorem. *The operator* $A-\beta_0$ *admits a densely defined inverse* $(A-\beta_0)^{-1}$.

Remark. It is shown in Yosida [18] that the semi-group in $C_0(R^N)$ associated with the N-dimensional Brownian motion admits a potential operator in his sense even in the *recurrent* cases, i.e., $N=1$ or 2 (see also Sato [13] and Hirsch [6], where one finds studies on the existence of potential operators associated with spatially homogeneous Markov

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1) We shall make use of the notation in Banach lattice. See, e.g., Chapter XII, 3 of Yosida [19].

2) Throughout the paper the mappings are all single-valued.