36. On the Boundary Value Problem for Elliptic System of Linear Differential Equations. II

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In this note we give the sketch of the proof of Theorem 1 in our preceding note [4] after giving another application (Example 4) of our theorem.

In this note we will use the same notations as in our previous note [4] and will not repeat their definition.

Further details of this note will appear elsewhere.

Example 4. In Example 2 of our previous note we treated only the Dirichlet problem. However Theorem 1, together with the results in Sato-Kawai-Kashiwara [5], [6], makes it possible to treat the so-called "non-elliptic boundary value problems" (cf. Hörmander [3], Egorov-Kondratev [1] and Sjöstrand [7]).

Let M be a real analytic manifold and N be a submanifold of M. For the sake of simplicity, we assume that N is of codimension 1 and defined by $\varphi(x)=0$. As in Example 2, $M=M_+\cup M_-\cup N, S_N^*M=N_+\cup N_-$. Then S_N^*X decomposes into three parts, namely, $S_\pm=q^{-1}(N_\pm)$ and $\sqrt{-1}S^*M$. Let $\mathcal{M}$ be an elliptic system of differential equations. For the sake of simplicity, we assume that $\mathcal{E}xt^j_{\mathcal{D}_M}(\mathcal{M},\mathcal{A}_M)=0$ for $j>0$. We denote by $\mathcal{S}$ the solution sheaf $\mathcal{H}om_{\mathcal{D}_M}(\mathcal{M},\mathcal{A}_M)$. We set $\mathcal{M}_\pm=p_*(\mathcal{P}_{Y\to X}\otimes_{\mathcal{D}_X}\mathcal{M}|_{S_\pm})$. Then $\mathcal{M}_\pm$ is a system of pseudo-differential equations on $\sqrt{-1}S^*N$, so that $\mathcal{M}_N=p_*(\mathcal{P}_{Y\to X}\otimes_{\mathcal{D}_X}\mathcal{M})$ is a direct sum of $\mathcal{M}_+$ and $\mathcal{M}_-$. Suppose that an admissible $\mathcal{P}_N$-subModule $\mathcal{N}$ of $\mathcal{M}_-$ is given so that the quotient sheaf $\mathcal{L}=\mathcal{M}_-/\mathcal{N}$ is also admissible. By Theorem 1, we have

$$(j_{+*}(\mathcal{S})/\mathcal{S})|_N=\pi_{N*}\mathcal{H}om_{\mathcal{P}_N}(\mathcal{M}_-,\mathcal{C}_N).$$

Hence we obtain an exact sequence

$$(1)\qquad 0\longrightarrow\pi_{N*}\mathcal{H}om_{\mathcal{P}_N}(\mathcal{L},\mathcal{C}_N)\longrightarrow(j_{+*}(\mathcal{S})/\mathcal{S})|_N \xrightarrow{B}\pi_{N*}\mathcal{H}om_{\mathcal{P}_N}(\mathcal{N},\mathcal{C}_N)\xrightarrow{\delta}\pi_{N*}\mathcal{E}xt^1_{\mathcal{P}_N}(\mathcal{L},\mathcal{C}_N).$$

The generalized boundary value problem means the problem to find a solution u of $\mathcal{M}$ defined on M_+ satisfying a boundary condition $Bu=\mu$, where μ is a given microfunction solution of $\mathcal{N}$. Therefore $\mathcal{E}xt^1_{\mathcal{P}_N}(\mathcal{L},\mathcal{C}_N)=0$ implies the existence of u for every μ and $\mathcal{H}om_{\mathcal{P}_N}(\mathcal{L},\mathcal{C}_N)=0$ implies the uniqueness of u (modulo the solutions defined on a neighborhood of N).