73. On a Semantics for Non-Classical Logics

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In [2] and [3], Ono showed some incompleteness results on two types of semantics for the intermediate predicate logics, that is, the algebraic and the Kripke-type. More precisely, he proved that there exist many intermediate predicate logics without characteristic sets of algebraic models, and that there exist those without characteristic Kripke models. This situation is more serious in the case of the modal predicate logics. In fact he proved the existence of a modal predicate logic having neither characteristic sets of algebraic models nor characteristic Kripke models.

Thus the existing semantical methods proved incomplete in the above sense. Therefore, some new type of semantics is required since the semantical method is indispensable for the study of logics.

This note proposes one of such semantics that contains the algebraic semantics as well as Kripke-type one as special cases. Our new semantics is obtained by combining these two types of semantics quite naturally.

Some applications of this semantics for the intermediate logics will be studied in a paper to appear in near future.

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§ 1. Semantics for the intermediate logics.

In this section we describe our semantics for the intermediate logics. Our basic language $\mathcal{L}$ is a usual one (see e.g. Ono [4]).

Definition 1.1. 1) *A pseudo-Boolean algebra P is called $\lambda^{<}$-complete, if there always exist $\bigcup_{t\in T} a_t$ and $\bigcap_{t\in T} a_t$ for any subset $\{a_t\}_{t\in T}$ of P such that $\overline{\overline{T}}<\lambda$.*

2) *A subalgebra P' of a pseudo-Boolean algebra P is said to be $\lambda^{<}$-complete, if for any subset $\{a_t\}_{t\in T}$ of P' such that $\overline{\overline{T}}<\lambda$ $\bigcup_{t\in T} a_t$, $\bigcap_{t\in T} a_t$ belong to P' whenever they exist in P.*

Definition 1.2. *By a model we mean a triple $(M, V; P)$ satisfying the following conditions;*

1) *M is a non-empty partially ordered set with the order relation $\leqslant_M$,*

2) *V is a mapping from M to the power set of some set such that $V(a)$ is non-empty for any $a \in M$ and $V(a)\subseteq V(b)$ if $a\leqslant_M b$,*

3) *P is a non-degenerate $\kappa(M, V)^{<}$-complete pseudo-Boolean*