72. On Banach-Steinhaus Theorem

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The theory of ranked space is a new and constructive method of the mathematical analysis, which has been investigated by K. Kunugi since 1954 [1]. We proved the closed graph theorem in ranked spaces with some conditions [4]. And now, in this note we shall prove the Banach-Steinhaus theorem in ranked spaces, whose neighbourhoods need not be open. Throughout this note, $g, f, \cdots$ will denote points of a ranked space, $U_i, V_i, \cdots$ neighbourhoods of the origin with rank i, $\{U_{r_i}\}, \{V_{r_i}\}, \cdots$ fundamental sequences of neighbourhoods with respect to the origin and $U_i(g), V_i(g), \cdots$ neighbourhoods of the point g with rank i.

Let a linear space E be a complete ranked space with indicator ω_0, which satisfies the following conditions.

(E, 1) (1) For any neighbourhood U_i, the origin belongs to U_i.

(2) For any U_i and V_j, there is a W_k such that $W_k \subsetneqq U_i \cap V_j$.

(3) For any neighbourhood U_i and for any integer n, there is an m such that $m \geqq n$ and $U_m \subsetneqq U_i$.

(4) The E is the neighbourhood of the origin with rank zero.

(E, 2) The following conditions are the modification of the Washihara's conditions [3].

(R, L_1) For any $\{U_{r_i}\}$ and $\{V_{r'_i}\}$, there is a $\{W_{r''_i}\}$ such that $U_{r_i} + V_{r'_i} \subsetneqq W_{r''_i}$.

(R, L_2)$'$ (1) For any $\{U_{r_i}\}$ and $\lambda > 0$, there is a $\{V_{r'_i}\}$ such that $\lambda U_{r_i} \subsetneqq V_{r'_i}$.

(2) For any $\{U_{r_i}\}$ and $\{\lambda_i\}$ with $\lim \lambda_i = 0$, $\lambda_i > 0$, there is a $\{V_{r'_i}\}$ such that $\lambda_i U_{r_i} \subsetneqq V_{r'_i}$.

(R, L_3) Let g be any point in E. For any $\{U_{r_i}\}$ there is a $\{V_{r'_i}(g)\}$, which is a fundamental sequence of neighbourhoods with respect to g, such that $g + U_{r_i} \subsetneqq V_{r'_i}(g)$ and conversely, for any $\{U_{r_i}(g)\}$ there is a $\{V_{r'_i}\}$ such that $U_{r_i}(g) \subsetneqq g + V_{r'_i}$.

(E, 3) For any neighbourhood U_i and for any $r > 0$, there exists some U_j such that $rU_i \supset U_j$.

(E, 4) For any neighbourhood $U_i(g)$ with respect to any g and for any $U_j(g)$ with $U_j(g) \subset U_i(g)$ and $j > i$, if $f \in U_j(g)$ there exists some neighbourhood U_k such that $f + U_k \subset U_i(g)$.