71. *Analyticity of Eigenvalues as Functions of Coupling Constant*

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Eigenvalues of the Hamiltonian in a quantum field theory can be considered as functions of the coupling constant. In the present paper, the analyticity of these functions is studied by using simple models.

1. Introduction. The total Hamiltonian H is written in the form $H=H^0+gH'$, where H^0 is the free Hamiltonian and gH' is the interaction term. In the eigenvalue equation

$$H\Psi=E\Psi \tag{1.1}$$

we call in question the analyticity of eigenvalues E which are functions of g. For this purpose, we use three simple models, the first two of them concern with the Lee model

$$V\rightleftarrows N\theta \tag{1.2}$$

and the third one is concerned with the reaction

$$\theta\rightleftarrows VN. \tag{1.3}$$

In the first case of reaction (1.2), all θ-particles have only one sort of momentum (*Case* 1). The second one of (1.2) is the usual Lee model (*Case* 2). In (1.3), θ-particles can have all sorts of momentum (*Case* 3). In these cases, the V- and N-particles have masses m_V and m_N, respectively, and are fixed in space. The rest mass of the Θ-particle is μ. The use of symbols V, N and a_k for the annihilation operators of V-, N- and θ-particles, respectively, is as usual [2]. In Case 1, symbol a is used in stead of a_k. The commutation or anticommutation relations among operators V, N, a_k etc. are also as usual [2]. In most cases, we assume the inequality $m_V>m_N+\mu$.

2. *Case* 1. We put $H^0=m_VV^*V+m_NN^*N+\omega a^*a$, and $H'=V^*Na+N^*Va^*$, where ω is a positive real constant. Let $u_V(0)$ and $u_V(1)$ be normalized eigenvectors of V^*V, and $u_N(0)$ and $u_N(1)$ be those of N^*N, and let $\mathfrak{H}_V$ and $\mathfrak{H}_N$ be two dimensional unitary spaces spanned by these sets of vectors, respectively. The infinite-dimensional Hilbert space spanned by normalized eigenvectors of a^*a is denoted by $\mathfrak{H}_\theta$. Then our basis space is the direct product of three spaces $\mathfrak{H}_V$, $\mathfrak{H}_N$ and $\mathfrak{H}_\theta$, i.e., $\mathfrak{H}(H^0)=\mathfrak{H}_V\otimes\mathfrak{H}_N\otimes\mathfrak{H}_\theta$. States in which the number of fermions is equal to one are written in the form

$$\Psi=\sum_{i+j=1}u_V(i)\otimes u_N(j)\otimes\phi_{ij}, \tag{2.1}$$