69. *Further Results for the Solutions of Certain Third Order Non-autonomous Differential Equations*

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1. Introduction. The differential equations considered here are

(1.1) $$\dddot{x}+\psi(t,x,\dot{x},\ddot{x})+\phi(t,x,\dot{x})+c(t)f(x)=p(t,x,\dot{x},\ddot{x})$$

(1.2) $$\dddot{x}+\psi(t,x,\dot{x},\ddot{x})+\phi(t,x,\dot{x})+c(t)f(x)=0$$

where ψ, ϕ, c, f and p are real valued functions. All solutions of (1.1) considered here are assumed real.

In [4] M. Harrow considered the behavior as $t\to\infty$ of solutions of the differential equation

(1.3) $$\dddot{x}+f(x,\dot{x},\ddot{x})\ddot{x}+g(x,\dot{x})+h(x)=p(t).$$

In [6] H. O. Tejumola considered the behavior as $t\to\infty$ of solutions of the differential equation

(1.4) $$\dddot{x}+f(t,\dot{x},\ddot{x})\ddot{x}+g(x,\dot{x})+h(x)=p(t,x,\dot{x},\ddot{x}).$$

Recently, in [3] T. Hara obtained some conditions under which all solutions of the equation

(1.5) $$\dddot{x}+a(t)f(x,\dot{x},\ddot{x})\ddot{x}+b(t)g(x,\dot{x})+c(t)h(x)=p(t,x,\dot{x},\ddot{x})$$

tend to zero as $t\to\infty$.

In [7], the author established conditions under which all solutions of the non-autonomous equation (1.1) tend to zero as $t\to\infty$.

In this note we investigate the asymptotic behavior of the solutions of the equation (1.1) under the condition weaker than that obtained in [3], [4], [6].

Many results have been obtained on the asymptotic properties of autonomous equations of third order and many of these results are summarized in [5].

2. Assumptions and Theorems. We shall state the assumptions on the functions ψ, ϕ, f, c and p appeared in the equation (1.1).

Assumptions.

(I) *$f(x)$ is a C^1-function in R^1, and $c(t)$ is a C^1-function in $I=[0,\infty)$.*

(II) *The function $\phi(t,x,y)$ is continuous in $I\times R^2$, and for the function $\phi(t,x,y)$ there exist functions $b(t), \phi_0(x,y)$ and $\phi_1(x,y)$ which satisfy the inequality*

$$b(t)\phi_0(x,y)\leqq\phi(t,x,y)\leqq b(t)\phi_1(x,y) \qquad \text{in } I\times R^2.$$

Moreover $b(t)$ is a C^1-function in I.