67. *Generalized Prime Elements in a Compactly Generated l-Semigroup. II*

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Let L be a cl-semigroup with the conditions (1), (2), (3), (4) and (∗) in [2]. Moreover we impose that *the compact generator system Σ of L is closed under multiplication.* The main purpose of this note is to define principal φ-components of elements in L by using φ-primes in [2], and to prove that every element of L is decomposed into their principal φ-components.

3. Principal φ-Components.

Let a be an element of L, and u an element of Σ. The *(left) φ-residual* $a:u$ of a by u is defined to be the supremum of the set of all elements x with $\varphi(u)\varphi(x)\leq a$, $x\in\Sigma$. We suppose throughout this note that *there is such elements x for any $a\in L$ and any $u\in\Sigma$.* For a, b in L, the *(left) φ-residual* $a:b$ of a by b is defined as infimum of the $a:u$, where u runs over $\Sigma(b)$. Then we can prove the following properties: 1) $a\leq a'$ implies $a:b\leq a':b$, $b:a\geq b:a'$ and 2) $(\bigcap_{i=1}^{n} a_i):b=\bigcap_{i=1}^{n}(a_i:b)$ for $a, a', a_i, b\in L$.

Now it is not so evident that $a:b\geq a$ for a, b in L. To prove this, it is sufficient to show that $(a:u)\cup a=a:u$ for $a\in L$ and $u\in\Sigma(b)$. Take an arbitrary element x of $\Sigma((a:u)\cup a)$. Then we can choose an element y of $\Sigma(a:u)$ with $x\leq y\cup a$. Since $y\leq\sup\{x'\in\Sigma\,|\,\varphi(u)\varphi(x')\leq a\}$, we can find a finite number of compact elements $x_1,\cdots,x_n$ such that $y\leq\bigcup_{i=1}^{n}x_i$ and $\varphi(u)\varphi(x_i)\leq a$. Then we have $x\leq\bigcup_{i=1}^{n}x_i\cup a\leq\bigcup_{i=1}^{n}\varphi(x_i)\cup a$, $\varphi(x)\leq\bigcup_{i=1}^{n}\varphi(x_i)\cup a$, and $\varphi(u)\varphi(x)\leq\bigcup_{i=1}^{n}\varphi(u)\varphi(x_i)\cup\varphi(u)a\leq a$. Therefore we obtain $(a:u)\cup a\leq a$, $(a:u)\cup a=a$.

(3.1) Definition. Let p be a maximal φ-prime element belonging to an element a of L. The *principal φ-component of a by p*, denoted by $a(p)$, is the supremum of all $a:s$, s runs over $\Sigma'(p)$, if $p\neq e$. If $p=e$, $a(p)$ is defined to be a.

(3.2) Lemma. *$a\leq a(p)$ and $a(p)$ is φ-related to a for any maximal φ-prime element p belonging to a.*

Proof. If $p=e$, the assertion is trivial. So we suppose that $p\neq e$. We want to prove that $a(p)\cup a=a(p)$. For the sake of this, take an arbitrary element x of $\Sigma(a(p)\cup a)$. Then since there is an element y

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