111. *On the Characterization of the Linear Partial Differential Operators of Hyperbolic Type*

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§1. **Introduction.** In this note we shall consider a linear partial differential operator $P(D)$ of degree m with real constant coefficients in n variables. By α we denote multi-indices, that is, n-tuples $(\alpha_1, \cdots \alpha_n)$ of non-negative integers and by $|\alpha|$ their sum, that is $|\alpha|=\sum_{j=1}^n \alpha_j$. With $D_j=-\sqrt{-1}\,\partial/\partial x_j$, we set $D^\alpha=D_1^{\alpha_1}\cdots D_n^{\alpha_n}$. Then the symbol $P(D)$ represents a differential operator $P(D)=\sum_{|\alpha|\leqq m} a_\alpha D^\alpha$ and if $(\xi_1, \cdots, \xi_n)\in C^n$, then $P(\xi)$ does the polynomial $P(\xi)=\sum_{|\alpha|\leqq m} a_\alpha \xi^\alpha$, $\xi^\alpha=\xi_1^{\alpha_1}\cdots\xi_n^{\alpha_n}$. This gives a one-to-one correspondence between polynomials and differential operators with constant coefficients. We shall call the operator $P(D)$ irreducible if the polynomial $P(\xi)$ is irreducible.

The aim of this note is to characterize the linear partial differential operator $P(D)$ by the support of the solution $u(x)\in C^\infty(R^n)$ of $P(D)u(x)=0$. If $u(x)$ satisfies $P(D)u(x)=0$, then $u(x)$ also satisfies $Q(D)P(D)u=0$ for arbitrary differential operator $Q(D)$. So we shall consider only irreducible linear partial differential operators.

Cohoon [1] proved the following theorem:

Theorem A. *There exists a nontrivial $u(x)$ in $C^\infty(R^n)$ such that $P(D)u(x)=0$ in R^n and such that the support of $u(x)$ is contained in $\{x\in R^n; |x_k|\leqq R$, for $k=1,2,\cdots n-1\}$ if and only if $P(D)$ is of the form*

$$P(D)=aD_n^m+\sum_{|\alpha|<m} b_\alpha D^\alpha$$

where a $(\neq 0)$ and b_α $(|\alpha|<m)$ are real constants.

Then we ask when there exists a nontrivial $u(x)$ in $C^\infty(R^n)$ such that $P(D)u(x)=0$ in R^n and such that the support of $u(x)$ is contained in $\{x\in R^n; |x_k|\leqq R$ for $k=1,\cdots,n-2$ and $(r|x_n|+R)^2-x_{n-1}^2\geqq 0\}$ for $r\geqq 0$. It is the purpose of this note to answer this question.

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§2. **Definitions and theorem.** By $P_m(D)$ we shall denote the principal part of $P(D)$. According to Hörmander [3] the operator $P(D)$ is called hyperbolic with respect to $N\in R^n$, if $P_m(N)\neq 0$ and if there is a constant τ_0 such that $P(\xi+i\tau N)\neq 0$, when $\tau<\tau_0$ and $\xi\in R^n$. For the principal part $P_m(D)$ the definition of hyperbolicity is particularly simple by the following theorem.