109. On the Global Existence of Real Analytic Solutions of Systems of Linear Differential Equations with Constant Coefficients

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In this note we shall give a necessary and sufficient condition for the global existence of real analytic solutions of systems of linear differential equations with constant coefficients. Recently L. Hörmander [1] has given a necessary and sufficient condition for single equations. Our result is a direct extension of Hörmander's.

1. Statements of the problem and the theorem. Let A be the ring of linear partial differential operators with constant coefficients in C^n. We may consider $A=C[\zeta_1,\cdots\zeta_n]$. Let M be an A module of finite type. Then it has a representation

$$0\longleftarrow M\longleftarrow A^t\overset{P(\zeta)}{\longleftarrow}A^s \tag{1.1}$$

where $P(\zeta)$ is a $t\times s$ matrix with elements in A, and we can consider the system of equations with constant coefficients ${}^tP(D)$ where $D=(D_1,\cdots,D_n)$ and $D_i=-\sqrt{-1}\partial/\partial x_i$. But such a representation is not unique and it is not ${}^tP$ but M that has an intrinsic meaning. Therefore we call M a system. (See V.P. Palamodov [2], M. Kashiwara [3], and M. Sato, T. Kawai and M. Kashiwara [4].)

Now let Ω be a convex domain in R^n and $\mathcal{A}(\Omega)$ be the set of real analytic functions in Ω. $\operatorname{Ext}^1_A(M,\mathcal{A}(\Omega))$ gives the obstruction of the global existence of real analytic solutions of inhomogeneous system ${}^tP(D)u=f$ where f satisfies compatibility conditions ${}^tQ(D)f=0$. Our problem is when

$$\operatorname{Ext}^1_A(M,\mathcal{A}(\Omega))=0 \tag{1.2}$$

is valid. Note that $\operatorname{Ext}^1_A(M,\mathcal{A}(\Omega))$ is independent of the choice of the representation (1.1).

Before stating our theorem let us recall some notions in commutative algebra. (See J. P. Serre [5] and Palamodov [2].) Let $0=M_1\cap\cdots\cap M_l$ be a primary decomposition of the submodule 0 in M. Ass (M) is the set of associated prime ideals of M, that is, the set of radicals $\boldsymbol{p}_i=r_M(M_i)=\{a\in A\,;\,\exists q\in Z_+\;\; a^qM\subset M_i\}$ $(i=1,\cdots,l)$. $V(M)=\{V_1,\cdots,V_l\}$ is the set of characteristic varieties, that is, the set of irreducible algebraic varieties associated to ideals in Ass (M).

Now we introduce the notion of components at infinity of charac-