104. A Typical Formal Group in K-Theory

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Typical formal groups were defined by Cartier [4] and used by Quillen [9] to decompose U-cobordism, localized at a prime p, into a direct sum of Brown-Peterson cohomologies with shifted degrees.

On the other hand, complex K-theory, localized at a prime p, was decomposed into $p-1$ factors by Adams [1] and Sullivan [11]. This decomposition is given in [1] with explicit idempotents. Its central factor inherits a multiplicative structure from K-theory so that we can expect a related formal group. In the present note the author observes that the desired formal group is in fact a typical group law with a simple nature.

As an application, using this typical formal group and a description of the polynomial basis of $BP^*(pt)$ (Theorem 1), we obtain a proof of Stong-Hattori theorem based on formal group techniques.

The details will appear elsewhere.

1. Typical formal groups. Let R be a commutative ring with unity and F a (one-dimensional) commutative formal group over R. A formal power series γ over R without constant term is called a *curve* over F. The addition $\gamma+_F\gamma'$ of two curves over F is defined by

$$(\gamma+_F\gamma')(T)=F(\gamma(T),\gamma'(T)).$$

With this addition the set C_F of all curves over F forms an abelian group. On C_F 3 kinds of operators are defined [4] by the following formulas:

i) $(\boldsymbol{f}_n\gamma)(T)=\sum_{k=1}^{n}{}_F\gamma(\zeta_k T^{1/n}), \qquad n\geqq 1,$

where $\zeta_k=\exp 2\pi k\sqrt{-1}/n$, the n-th roots of unity;

ii) $(\boldsymbol{v}_n\gamma)(T)=\gamma(T^n), \qquad n\geqq 1;$

iii) $([a]\gamma)(T)=\gamma(aT), \qquad a\in R.$

Operators $\boldsymbol{f}_n$ are called *Frobenius operators* and particularly important. These 3 kinds of operators satisfy certain universal relations [4], and we treat C_F as an operator-module. A curve γ_0 defined by $\gamma_0(T)=T$ will be regarded as the one of the basic curves.

Some functorialities of these operator-modules should be observed. Let F and G be formal groups over R and $\varphi: F\rightarrow G$ a homomorphism, i.e., a curve over G satisfying

$$\varphi\circ F=G\circ(\varphi\times\varphi).$$