155. *Incompleteness of Semantics for Intermediate Predicate Logics. I*

Kripke's Semantics

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Kripke models have been introduced in [2] for the intuitionistic logic, but in [3] we have studied their basic properties as models for intermediate propositional logics. We presented there the following problem.

Has every intermediate propositional logic a characteristic Kripke model?

Though it is important, e.g. in connection with the finite model property ([4]), it remains unsolved. Instead of this, we will show in the present paper that there exists an intermediate predicate logic having no characteristic Kripke models. In this sense, we can say that Kripke's semantics for intermediate predicate logics is incomplete. Similar incompleteness results for algebraic semantics will appear in the subsequent paper.

In order to abbreviate definitions, we use some of the terminology in Church [1]. We identify the word *predicate logics* with the word *pure functional calculi of first order.*

At first, we fix a language of pure functional calculus of first order. LK (and LJ) denote the pure classical (and intuitionistic) functional calculus of first order. The definition of intermediate predicate logics in general sense is given in [5], but here we only deal with finitely axiomatizable ones. Let A be a formula provable in LK. Then by $LJ+A$, we mean the intermediate predicate logic obtained by adding an axiom scheme A to LJ.

Definition 1. A pair (M, V) is called a Kripke model if M is a nonempty set with a partial order (denoted as $\leq$) and V a function from M to the power set of a set such that $V(a)\subseteq V(b)$ if $a\leq b$ and $V(a)\neq\emptyset$ for any $a\in M$.

A valuation W on (M, V) is a function which takes one of truth values $\{t, f\}$ as its value for a pair (A, a) of a formula A and an element a of M and whose values are determined by the rules in [2]. A formula A is said to be *valid* in (M, V), if $W(A^*, a)=t$ for any valuation W on (M, V) and any $a\in M$, where A^* is the closure of A. We write the set