178. *A Generalization of the Closed Graph Theorem*

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S. Banach ([3]) proved the closed graph theorem: every linear mapping with a closed graph from any complete metrizable topological vector space into another one is continuous, and also proved the open mapping theorem: every continuous linear mapping from any complete metrizable topological vector space onto another one is open. Many generalizations of these theorems have been tried. Recently, N. Adasch ([1] and [2]) and W. Robertson ([5]) introduced two classes of topological vector spaces: "infra-s-Raum" or "K_r-complete space" and "s-Raum" or "K-complete space" by N. Adasch or W. Robertson, respectivery. They proved that the former space is the (locally convex) Hausdorff topological vector space satisfying a minimal condition under which the closed graph theorem holds for all linear mappings from any ultrabarrelled (barrelled) spaces, and that the later space is the (locally convex) Hausdorff topological vector space satisfying a minimal condition under which the open mapping theorem holds for every linear mapping with a closed graph onto any ultrabarrelled (barrelled) space. N. Adasch ([2]) also proved that a Hausdorff topological vector space $F[\eta_0]$ is an infra-s-Raum if and only if η^{ut} coincides with η_0^{ut} for every Hausdorff vector topology η coarser than η_0, where $\eta^{ut}(\eta_0^{ut})$ is the ultrabarrelled topology associated with the topology $\eta(\eta_0)$. (Hereafter we assume that a notation $F[\eta]$ means the topological vector space F with the vector topology η.)

S. O. Iyahen ([4]) celled a topological vector space E **-inductive limit* of a family $\{E_\alpha\}$ of topological vector spaces by mappings $\{u_\alpha\}$, where for each α, u_α maps E_α into E and the union of the subspaces $u_\alpha(E_\alpha)$ spans E, if and only if the topology of E is the finest vector topology making every u_α continuous.

We shall generalize the Banach's theorems furthermore by these results. For locally convex spaces we have the same results, which are proved in this paper for general topological vector spaces, by using the concept of the inductive limit instead of the one of the *-inductive limit.

In this paper we consider that a property (p) on topological vector

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