174. On the Structure of Certain Types of Polarized Varieties

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1. This is a report on our recent results on a study of structures of polarized varieties. Details will be published elsewhere.

In this note we mean by an algebraic variety a complex space associated with an irreducible, reduced and proper $\boldsymbol{C}$-scheme. We fix our notation.

$c_j(E)$: the j-th Chern class of a vector bundle E,
$\boldsymbol{P}(E)$: the projective bundle associated with E,
$\boldsymbol{L}(E)$: the tautological line bundle on $\boldsymbol{P}(E)$,
E^*: the dual vector bundle of E,
$|F|$: the complete linear system of Cartier divisors associated with a line bundle F,
B_sL: the set of base points of a linear system L,
$[W]$: the natural integral base of $H_{2n}(W;\boldsymbol{Z})$ where W is a variety of dimension n,
K_M: the canonical line bundle on a manifold M.

Let F be an ample line bundle on a variety V. We call such a pair (V,F) a polarized variety. In addition if V is non-singular we call (V,F) a polarized manifold. We say that (V_1,F_1) is isomorphic to (V_2,F_2) and write $(V_1,F_1)\cong(V_2,F_2)$ if there is a biholomorphic mapping $f:V_1\to V_2$ such that $F_1=f^*F_2$. We define the following invariants of a polarized variety (V,F) of dimension n:
$d(V,F)=F^n=(c_1(F))^n[V]$,
$\Delta(V,F)=\dim V+d(V,F)-\dim H^0(V,\mathcal{O}_V(F))$,
and if V is non-singular, we define
$g(V,F)=(K_V+(n-1)F)F^{n-1}/2+1$.
The importance of $\Delta(V,F)$ is illustrated by the following fact.

Lemma A. *Let (V,F) be a polarized variety. Then $\dim B_s|F| <\Delta(V,F)$, where $\dim\emptyset$ is defined to be -1. In particular $\Delta(V,F)\geqq 0$ for every polarized variety.*

In section 2 we give a complete classification of polarized manifolds with $\Delta=0$. In section 3 we give certain structure theorems concerning polarized manifolds with $\Delta=1$, and classify such manifolds except the case in which $d=5,6$ and $\dim M=3$.

Our proof by induction with respect to the dimension of the