170. *Maximum Principles for Implicit Parabolic Equations*

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In [6], Nirenberg derived a strong maximum principle for second order linear parabolic equations. This result was extended by Besala [2] to nonlinear parabolic equations of the form

$$(*)\qquad u_t=f(t,x,u,u_x,u_{xx}),$$

where $x=(x_1,\cdots,x_n)$, $u_t=\partial u/\partial t$, $u_x=(\partial u/\partial x_i)_{i=1}^n$ and $u_{xx}=(\partial^2 u/\partial x_i\partial x_j)_{i,j=1}^n$.

On the other hand, Picone [7] and Krzyżański [4] established a maximum principle in unbounded domains which was particularly useful to the study of the Cauchy problem for second order linear parabolic equations. An extension of this principle to nonlinear equations of the form $(*)$ was given by Besala [1].

The purpose of this paper is to generalize the above mentioned results of Besala to the implicit parabolic equation

$$(1)\qquad F(t,x,u,u_t,u_x,u_{xx})=0.$$

Let D be a domain in the $(n+1)$-dimensional Euclidean space R^{n+1} of points (t,x). For each fixed point $(t^0,x^0)\in D$ we define $S_D^+(t^0,x^0)[S_D^-(t^0,x^0)]$ to be the set of all points $(t,x)\in D$ which can be joined to (t^0,x^0) by a upward [downward] directed broken line contained in D, with (t^0,x^0) as initial point and (t,x) as endpoint.

Consider a function $F(t,x,z,p,Q,R)$ defined for all $(t,x)\in D,z,p$, $Q=(q_i)_{i=1}^n$ and $R=(r_{ij})_{i,j=1}^n$. The function $F(t,x,z,p,Q,R)$ is said to belong to the class $\mathcal{P}(D)$ if there exist positive constants κ and τ such that

$$(2)\qquad F(t,x,z,p,Q,R)-F(t,x,z,\tilde{p},Q,\tilde{R})\geqq \kappa\sum_{i=1}^n(r_{ii}-\tilde{r}_{ii})+\tau(\tilde{p}-p)$$

for all $(t,x)\in D,z,p,\tilde{p}$ with $p\leqq\tilde{p}$, Q, and symmetric matrices $R=(r_{ij})$, $\tilde{R}=(\tilde{r}_{ij})$ such that $R-\tilde{R}$ is positive semidefinite.

First, we shall prove a strong maximum principle for equation (1) which extends a recent result of Besala [2].

Theorem 1. *Assume that the function $F(t,x,z,p,Q,R)$ belongs to the class $\mathcal{P}(K)$ for any compact subset K of D, and that there exist positive constants L_0,L_1,L_2 and L_3 such that*

$$(3)\qquad \begin{aligned}&|F(t,x,z,p,Q,R)-F(t,x,\tilde{z},\tilde{p},\tilde{Q},\tilde{R})|\\ &\quad\leqq L_0|z-\tilde{z}|+L_1|p-\tilde{p}|+L_2\sum_{i=1}^n|q_i-\tilde{q}_i|+L_3\sum_{i,j=1}^n|r_{ij}-\tilde{r}_{ij}|\end{aligned}$$

for all $(t,x)\in K,z,\tilde{z},p,\tilde{p},Q,\tilde{Q},R$ and $\tilde{R}$.

Let $u(t,x)$ and $v(t,x)$ be continuous and continuously differentiable