14. *On the Asymptotic Behavior of Resolvent Kernels and Spectral Functions for Some Class of Hypoelliptic Operators*

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1. Introduction. For hypoelliptic operators with constant coefficients studies on asymptotic behavior of their spectral functions were done by Nilsson [10], Gorčakov [6] and Friberg [4] (cf. [15]). For the case of operators with variable coefficients Nilsson [11] has studied this problem for formally hypoelliptic operators and Smagin [12] has done that for some class of hypoelliptic operators for which a complex power can be defined. In this paper we shall anounce some results on that problem and asymptotic distribution of eigenvalues for the case of variable coefficients by a method of pseudo-differential operators (cf. [7], [8]). Let $P=P(x,D)=\sum_{|\alpha|\le m} a_\alpha(x)D^\alpha$ be a formally self-adjoint linear partial differential operator with its domain $C_0^\infty(\Omega)$, where $x=(x_1,\cdots,x_n)$ is a point of real n-space R_x^n, $\alpha=(\alpha_1,\cdots,\alpha_n)$ is a multi-index of which length $|\alpha|=\alpha_1+\cdots+\alpha_n$ and D^α or $D_x^\alpha=(-i\partial/\partial x_1)^{\alpha_1}\cdots(-i\partial/\partial x_n)^{\alpha_n}$. The coefficients $a_\alpha(x)$ are supposed to be in $\mathscr{B}(\Omega)$ in the notation of L. Schwarz for an open set Ω in R_x^n. For $\xi\in R^n$ we denote $|\xi|=(\xi_1^2+\cdots+\xi_n^2)^{1/2}$, $\langle\xi\rangle=1+|\xi|$ and $\xi^\alpha=\xi_1^{\alpha_1}\cdots\xi_n^{\alpha_n}$. For $P(x,\xi)$ $\sum_{|\alpha|\le m} a_\alpha(x)\xi^\alpha$ we set $P_{(\beta)}^{(\alpha)}(x,\xi)=D_\xi^\alpha(iD_x)^\beta P(x,\xi)$.

2. A class of hypoelliptic operators, theorems. We assume the followings on $P(x,\xi)$: this is written in the sum $P(x,\xi)=p_0(x,\xi)+p_1(x,\xi)$ and for any $x\in\Omega$ and α and β there exist positive constants $C_{x,\alpha,\beta}$, C_x and A_x such that

$$|p_{0(\beta)}^{(\alpha)}(x,\xi)|\le C_{x,\alpha,\beta}\,|p_0(x,\xi)|^{1-\rho|\alpha|+\delta|\beta|} \tag{2.1}$$

$$|p_{1(\beta)}^{(\alpha)}(x,\xi)|\le C_{x,\alpha,\beta}\,|p_0(x,\xi)|^{1-\rho(|\alpha|+1)+\delta(|\beta|+1)} \tag{2.1$'$}$$

for $|\xi|\ge A_x$, where ρ and δ are some constants depending only on $P(x,\xi)$ and satisfying $0\le\rho<\delta\le 1/m$, and

$$|p_0(x,\xi)|\ge C_x\,|\xi|^{m'},\quad 0<m'\le m\quad \text{for } |\xi|\ge A_x, \tag{2.2}$$

$$m'>n. \tag{2.3}$$

We remark that (2.3) can be removed by considering a power of $P(x,D)$. We assume further that $C_{x,\alpha,\beta}$, C_x and A_x are bounded when x is in a compact subset of Ω. We consider the case in which $p_0(x,\xi)$ is taken real because of the self-adjointness of $P(x,D)$, and assume $p_0(x,\xi)\to+\infty$ as $|\xi|\to\infty$. We have proved in [13] the following: