9. *On the Completions of Maps*

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In this paper all spaces are assumed to be completely regular T_2. Let f be a continuous map from a space X into a space Y. As is well known, there exists its extension $\beta(f):\beta(X)\to\beta(Y)$, where $\beta(S)$ denotes the Stone-Čech compactification of a space S. Furthermore, it is known that $\beta(f)$ carries $\mu(X)$ into $\mu(Y)$ and $\upsilon(X)$ into $\upsilon(Y)$ ([14], [3]), where $\mu(X)$ is the topological completion of X (that is, the completion of X with respect to its finest uniformity μ) and $\upsilon(X)$ is the realcompactification of X. We denote the restriction maps $\beta(f)|\mu(X)$ and $\beta(f)|\upsilon(X)$ by $\mu(f)$ and $\upsilon(f)$ respectively.

The purpose of this paper is to study the relations between f and $\mu(f)$ (or $\upsilon(f)$).

We note first that $\mu(f):\mu(X)\to\mu(Y)$ and $\upsilon(f):\upsilon(X)\to\upsilon(Y)$ are not necessarily perfect even if $f:X\to Y$ is perfect. A continuous map f from a space X onto a space Y is called a quasi-perfect (perfect) map if f is a closed map such that $f^{-1}(y)$ is countably compact (resp. compact) for each $y\in Y$.

Example. Let Y be a pseudo-compact space such that the preimage X of Y under a perfect map f is not pseudo-compact ([4, Example 4.2]). Then both $\mu(f):\mu(X)\to\mu(Y)$ and $\upsilon(f):\upsilon(X)\to\upsilon(Y)$ are not perfect, since $\mu(X)$ and $\upsilon(X)$ are not compact, while $\mu(Y)$ and $\upsilon(Y)$ are compact (cf. [14], [3]).

In view of these results, it is significant to study under what conditions $\mu(f)$ (or $\upsilon(f)$) is perfect.

Theorem 1. *If $f:X\to Y$ is an open quasi-perfect map, then $\mu(f):\mu(X)\to\mu(Y)$ and $\upsilon(f):\upsilon(X)\to\upsilon(Y)$ are open perfect.*

To prove this theorem, we use the following lemmas.

Lemma 2 (Zenor [17]). *Let $C(X)$ be the space of all the non-empty compact sets in a space X with the finite topology. If X is topologically complete, so is $C(X)$.*

The finite topology of $C(X)$ is defined as follows: For any finite number of open sets $\{U_1,\cdots,U_n\}$ of X, we set $[U_1,\cdots,U_n]=\{K\in C(X)\,|\,K\subset\bigcup_{i=1}^{n}U_i,\ K\cap U_i\neq\emptyset$ for $i=1,\cdots,n\}$. As an open base of $C(X)$ we take all such sets. It is well known that if X is completely regular then so is $C(X)$ (Michael [12]).