8. *Paracompactness of Topological Completions*

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1. Introduction. All spaces are assumed to be completely regular T_2 unless otherwise specified. This paper is mainly concerned with paracompactness of the completion $\mu(X)$ of a space X with respect to its finest uniformity μ. Such completion of a space X is called the topological completion of X (or the completion in the sense of Dieudonné). Following Morita [12], a space X is said to be pseudo-paracompact (resp. pseudo-Lindelöf etc.) if $\mu(X)$ is paracompact (resp. Lindelöf etc.). Since for any M-space X $\mu(X)$ is a paracompact M-space ([12]), every M-space is pseudo-paracompact.

The purpose of this paper is to study some properties of pseudo-paracompact spaces. The details will be published elsewhere.

2. Characterizations of pseudo-paracompact spaces. An open covering $\mathfrak{O}=\{O_\alpha\}$ of a space X is said to be extendable to $\mu(X)$ if there exists an open covering $\tilde{\mathfrak{O}}=\{\tilde{O}_\alpha\}$ of $\mu(X)$ such that $O_\alpha=\tilde{O}_\alpha \cap X$ for each α. We note that every normal open covering of X is extendable to $\mu(X)$ as a normal open covering (cf. [9, (I) Lemma 8 and (II) Lemma 1]).

Now let $\{\mathfrak{U}_\lambda \mid \lambda \in \Lambda\}$ be the set of all the normal open coverings of a space X. A filter $\mathfrak{F}=\{F_\alpha\}$ in X is said to be weakly Cauchy with respect to the uniformity μ if for any $\lambda \in \Lambda$ there exists $U \in \mathfrak{U}_\lambda$ such that $U \cap F_\alpha \neq \phi$ for every α. In other words, a filter $\mathfrak{F}$ is weakly Cauchy if for any $\lambda \in \Lambda$ there exists a stronger filter $\mathfrak{F}_\lambda$ than $\mathfrak{F}$ such that $L \subset U$ for some $U \in \mathfrak{U}_\lambda$ and $L \in \mathfrak{F}_\lambda$. We state first the necessary and sufficient conditions for a space X to be pseudo-paracompact, some of which are the modifications of Corson's theorem [1] for the characterizations of paracompact spaces.

Theorem 2.1. *For a space X, the following conditions are equivalent.*

(a) *X is pseudo-paracompact.*

(b) *Every open covering of X which is extendable to $\mu(X)$ is a normal covering.*

(c) *The product of X with every compact space is pseudo-normal.*

(d) *Every weakly Cauchy filter in X with respect to μ is contained in some Cauchy filter with respect to μ.*

(e) *If $\mathfrak{F}$ is a filter in X such that the image of $\mathfrak{F}$ has a cluster*