5. *The Asymptotic Eigenvalue Distribution for Non-smooth Elliptic Operators*

By Hideo TAMURA
Department of Mathematics, Nagoya University

(Comm. by Kôsaku YOSIDA, M. J. A., Jan. 12, 1974)

1. Introduction.

The purpose of this note is to study the asymptotic eigenvalue distribution for the following equation

(1.1) $$Au+ru=\lambda pu \qquad r\geq 0.$$

Here A is a positive elliptic differential operator with constant coefficients defined on R^n and $p(x)$ is a positive function. When A is a homogeneous elliptic operator with a non-smooth $p(x)$, the distribution of the eigenvalues of (1.1) was discussed in Birman and Solomjak [3], Birman and Borzov [4] and Rosenbljum [5]. In this note we will study the asymptotic distribution including the case that A is an inhomogeneous operator. The obtained results can be applied to the operator with a large parameter $h>0$

(1.2) $$Au-hp(x)u=\mu u.$$

In fact, it was shown in Birman [2] that the number of negative eigenvalues less than r of equation (1.2) coincides with the number of eigenvalues less than h of equation (1.1).

Only the theorems and an outline of proofs are presented here and details will be published elsewhere.

2. Main Theorems.

Let $A(D)=\sum_{|\alpha|\leq m} a_\alpha D^\alpha$ be an elliptic operator with constant coefficients defined on R^n. We suppose that:

(i) $A(\xi)\geq 0 \qquad \text{for } \xi\in R^n$;

(ii) $\xi=0$ is the only zero of $A(\xi)$ of even order $m_0\leq m$.

The principal part of $A(D)$ is denoted by $A_0(D)$.

We denote by $K(l,a)$ $(l>0, a>0)$ the set of functions $p(x)$ which satisfy the following conditions:

(i) $p(x)$ is decomposed into $p(x)=p_1(x)+p_2(x)$;

(ii) $p_1(x)$ is a positive smooth function with $\lim_{|x|\to\infty} |x|^l p_1(x)=a$;

(iii) $p_2(x)$ is a nonnegative function with compact support;

(iv) $p_2(x)\in L_p$, where $p=1$ if $m\geq n$ and $p>\dfrac{n}{m}$ if $m<n$.

Let $N_r(\lambda)$ be the number of eigenvalues less than λ of equation (1.1).