1. *On the Degenerate Oblique Derivative Problems*

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§ 1. **Introduction and results.** In this note we give *a priori* estimates and existence theorems for the degenerate oblique derivative problems, which will be formulated below. We first reduce the given boundary value problems to the pseudo-differential equations on the boundary with the aid of suitable boundary value problems which are well studied, and next apply Melin's theorem (see [5], Theorem 3.1) to the pseudo-differential equations on the boundary.

Let Ω be a bounded domain in R^n, and we assume that $\Omega \cup \partial\Omega$ is a C^∞-manifold with boundary. Let $a(x)$, $b(x)$ and $c(x)$ be real-valued functions $\in C^\infty(\partial\Omega)$, $\boldsymbol{n}$ be the unit exterior normal to $\partial\Omega$ and $\boldsymbol{\nu}$ be a real C^∞-vector field on $\partial\Omega$.

Now we consider, for $\lambda > 0$, the degenerate oblique derivative problem:

$$\text{(I)} \qquad \begin{cases} (\lambda - \Delta)u = f & \text{in } \Omega, \\ a(x)\dfrac{\partial u}{\partial \boldsymbol{n}} + b(x)\dfrac{\partial u}{\partial \boldsymbol{\nu}} + c(x)u = 0 & \text{on } \partial\Omega, \end{cases}$$

under the following assumptions:

(1) $a(x) \geqq 0$.

(2) The set $S = \{x \in \partial\Omega\,;\, a(x) = 0\}$ is an $(n-2)$-dimensional C^∞-manifold.

(3) $\boldsymbol{\nu}$ is transversal to S in $\partial\Omega$.

(4) $c(x) > 0$ on the set $\{x \in \partial\Omega : a(x) = 0\}$.

(5) Along the integral curve $x(t, x_0)$ of $\boldsymbol{\nu}$ passing $x_0 \in S$ when $t=0$, $a(x(t, x_0))$ has a zero of finite order k at $t=0$, and $b(x(t, x_0))$ has a zero of finite order l at $t=0$, where k and l are independent of x_0.

Remark 1. In the case where $b(x) \neq 0$ on S, our problem is the oblique derivative problem which has been already treated by several authors and we can remove the assumption (4) (see [2] and [6]). In the case where $b(x) \equiv 0$, our problem was treated by S. Itô (see [3]) and we can also remove the assumptions (2) and (5) (see the proof below).

For each real s, we denote by $H_s(\Omega)$ and $H_s(\partial\Omega)$ the usual Sobolev spaces on Ω and $\partial\Omega$ respectively, and by $\|\ \|_{s,\Omega}$ and $\|\ \|_{s,\partial\Omega}$ norms in these spaces.

Theorem 1. *Assume that* $l \geqq k$ *and the assumptions* (1), (2), (4) *and* (5) *hold. Then there is a positive constant* C *such that, for* $u \in L^2(\Omega)$