28. *Examples of Foliations with Non Trivial Exotic Characteristic Classes*

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1. Introduction. In [1], R. Bott has defined the exotic characteristic classes for foliations as follows:

Let $q \geqq 1$ be an integer.

First, a cochain complex (WO_q, d) is defined. Let $\boldsymbol{R}[c_1, \cdots, c_q]$ denote the graded polynomial algebra over $\boldsymbol{R}$ generated by the elements c_i with degree $2i$. Set

$$\boldsymbol{R}_q[c_1, \cdots, c_q] = \boldsymbol{R}[c_1, \cdots, c_q]/\{\phi\,;\ \deg(\phi) > 2q\}.$$

Let $E(h_1, h_3, \cdots, h_r)$ denote the exterior algebra over $\boldsymbol{R}$ generated by the elements h_i with degree $2i-1$, where r is the largest odd integer $\leqq q$. Then, as a graded algebra over $\boldsymbol{R}$

$$WO_q = \boldsymbol{R}_q[c_1, \cdots, c_q] \otimes E(h_1, h_3, \cdots, h_r)$$

and a unique antiderivation of degree 1

$$d : WO_q \to WO_q$$

is defined by requiring

$$d(c_i) = 0, \qquad i = 1, \cdots, q$$
$$d(h_i) = c_i, \qquad i = 1, 3, \cdots, r.$$

Secondly, given a C^∞-smooth codimension q foliation $(N, \mathcal{F})$ on an oriented manifold N without boundary, a homomorphism of cochain complexes

$$\lambda_{(N,\mathcal{F})} : WO_q \to A_C^*(N)$$

is defined, where $A_C^*(N)$ denotes the space of complex smooth forms on N. We used the notation $\lambda_{(N,\mathcal{F})}$ in place of λ_E of Bott [1]. Here the homomorphism $\lambda_{(N,\mathcal{F})}$ depends only on the choices of two connections on the normal bundle of the foliation $(N, \mathcal{F})$ called metric and basic.

In cohomology, $\lambda_{(N,\mathcal{F})}$ induces a homomorphism of graded $\boldsymbol{R}$-algebras

$$\lambda_{(N,\mathcal{F})}^* : H^*(WO_q) \to H^*(N\,; \boldsymbol{C})$$

which does not depend on the choices of the above connections.

The elements of $\lambda_{(N,\mathcal{F})}^*(H^*(WO_q))$ are called the exotic characteristic classes for the foliation $(N, \mathcal{F})$.

In this paper, we construct the examples of foliations with non trivial exotic characteristic classes, that is,

Theorem. *For any integer $q \geqq 1$, there exists a C^∞-smooth*