25. *On the Bauer Simplexes and the Uniform Algebras*

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1. A Bauer simplex is a simplex whose set of extreme points is closed. We consider in this note when the state space of a uniform algebra is a Bauer simplex (Proposition 2). The result is applied to the tensor product $A_1\hat{\otimes}A_2$ of uniform algebras A_1 and A_2, and we show that all the Gleason parts of at most one of A_1 and A_2 must be trivial if $A_1\hat{\otimes}A_2$ is u.r.m. (i.e. every maximal measure representing a complex homomorphim of $A_1\hat{\otimes}A_2$ is unique).

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2. We shall make use of the definitions and the notions of [1]. Let K be a compact convex subset of some locally convex space and let $\mathcal{A}(K)$ denote the Banach space of real valued continuous affine functions on K. The set of extreme points of K is denoted by ∂K.

First we give a slight generalization of Bauer's theorem (cf., [1, p. 105]).

Proposition 1. *Let K be a compact convex set and E a real complete locally convex space. Then K is a Bauer simplex if and only if every continuous map f of ∂K into E has an extension to a continuous affine map of K into E. In particular, if E is a Banach space, then this extension can be made norm preserving.*

Proof. Assume that K is a Bauer simplex. Then ∂K is a closed subset of K. Hence $f(\partial K)$ is a compact subset of E. Since E is complete, the closed convex hull F of $f(\partial K)$ is a compact convex subset. By Bauer's theorem, every boundary measure annihilating $\mathcal{A}(K)$ is null, and from Alfsen [2, Corollary to Theorem A] there exists a continuous affine map $\tilde{f}$ of K into F such that $\tilde{f}|_{\partial K}=f$. If E is a Banach space, then $\|f\|_{\partial K}=\|\tilde{f}\|_K$. The converse statement is reduced to Bauer's theorem by considering a one-dimensional subspace of E. This completes the proof.

3. Let A be a uniform algebra on a compact Hausdorff space X. We denote by $\partial_A X$, $\Gamma(A)$, $M(A)$ and $S(A)$ the Choquet boundary, the Šilov boundary, the maximal ideal space and the state space of A respectively. A is called a Dirichlet algebra if $\operatorname{Re} A|_{\Gamma(A)}$ is dense in $C_R(\Gamma(A))$.