22. *Uniqueness in the Cauchy Problem for Partial Differential Equations with Multiple Characteristic Roots*

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1. Introduction. We are concerned with the uniqueness theorem in the Cauchy problem for the following type of partial differential equations:

$$Pu \equiv \partial_t^m u + \sum_{|\alpha|+j\leqslant m} a_{\alpha,j}(x,t)\partial_x^\alpha \partial_t^j u = 0, \qquad (x \in R^l).$$

Here we assume $a_{\alpha,j}(x,t)$ are sufficiently smooth functions. In the case where the characteristic roots are simple and the coefficients $a_{\alpha,j}(x,t)$ $(|\alpha|+j=m)$ are all real, A. P. Calderón [1] proved the uniqueness theorem in 1958. When (x,t) is two-dimensional, T. Carleman [2] obtained the same result as early as 1938. S. Mizohata [6] proved the uniqueness in the case of elliptic type of order 4 with smooth characteristic roots. Many authors have studied the uniqueness with at most double smooth characteristic roots ([3], [5], etc.). Then a study for elliptic type with triple characteristic roots, was made by K. Watanabe [10], under the assumption that the multiplicity of the characteristic roots is constant.

The purpose of this note is to announce with a short proof a result on the uniqueness theorem for operators with multiple characteristic roots. A forthcoming article will give a detailed proof. Let us consider the following type of operator:

$$P = P_p(x,t;\partial_x,\partial_t)^m + P_{mp-1}(x,t;\partial_x,\partial_t) + R(x,t;\partial_x,\partial_t),$$

where $m \geqslant 2$ and $p \geqslant 1$. Here we assume that, 1) P_p is a homogeneous partial differential operator of order p with real coefficients, continuously differentiable up to order $l+\max\{mp,6\}$. Moreover its characteristic roots $\{\lambda_j(x,t;\xi)\}_{1\leqslant j\leqslant p}$ of $P_p(x,t;\xi,\lambda)=0$ are distinct for all real $\xi(\neq 0)$, 2) P_{mp-1} is a homogeneous partial differential operator of order $mp-1$ with real coefficients belonging to $C^{l+\max\{mp-1,5\}}$, 3) R is a partial differential operator of order at most $mp-2$, with bounded measurable coefficients.

Let $\{\lambda_j(x,t;\xi)\}_{1\leqslant j\leqslant p}$ be the characteristic roots of P_p. We introduce the following conditions.

(A) $\quad P_{mp-1}(0,0;\xi,\tau)|_{\tau=\lambda_j(0,0;\xi)} \neq 0 \quad$ for all $\xi \in R^l-\{0\} \quad (1\leqslant j\leqslant p)$

(B_1) $\quad P_{mp-1}(x,t;\xi,\tau)|_{\tau=\lambda_j(x,t;\xi)} \equiv 0 \quad$ for all $(x,t,\xi) \in U\times(R^l-\{0\})$ $\quad (1\leqslant j\leqslant p)$