49. *On Fixed Point Theorem*

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In this paper we shall prove a fixed point theorem by the method of ranked space. The linear operator in the following theorem is not necessarily continuous. Throughout this note, $g, f, x, y, z, \cdots$ will denote points of a ranked space, $U_i, V_i, \cdots$ neighbourhoods at the origin with rank i and $\{U_{r_i}\}, \{V_{r_i}\}, \cdots$ fundamental sequences of neighbourhoods with respect to the origin. Let a linear space E be a ranked space with indicator ω_0, which satisfies the following conditions:

(E, 1)
(1) For any neighbourhood U_i, the origin belongs to U_i.
(2) For any neighbourhood U_i, and for any integer n, there is an m such that $m \geqq n$ and $U_m \subseteqq U_i$.
(3) The space E is the neighbourhood at the origin with rank zero.

Furthermore we define $g+U_i$ as a neighbourhood at point g with rank i. Then the space E is called a pre-linear ranked space. Moreover the space E having the following conditions (E, 2) and (E, 3), is called a linear ranked space.

(E, 2) The following conditions are the modification of the Washihara's conditions [3].

(R, L_1) For any $\{U_{r_i}\}$ and $\{V_{r_i'}\}$, there is a $\{W''_{r_i}\}$ such that

$$U_{r_i} + V_{r_i'} \subseteqq W_{r_i''}.$$

(R, L_2)″ For any $\{U_{r_i}\}$ and any $\lambda > 0$, there are a $\{U_{r_i'}\}$, all of whose members belong to $\{U_{r_i}\}$ and a natural number j such that

$$\lambda U_{r_i} \subseteqq U_{r_i'} \quad \text{for all } i \ (i \geqq j).$$

(E, 3) For any neighbourhood U_i and any λ $(0 \leqq \lambda \leqq 1)$, $\lambda U_i \subseteqq U_i$.

Definition 1 (T_1-space). A pre-linear ranked space E is called a T_1-space if for any g, f $(g \neq f, g \in E, f \in E)$ and any fundamental sequence at the origin $\{U_{r_i}\}$ there exists some U_{r_j} belonging to $\{U_{r_i}\}$ such that $g+U_{r_j} \not\ni f$.

Definition 2 (T_2-space). A pre-linear ranked space E is called a T_2-space if for any g, f $(g \neq f, g \in E, f \in E)$ and any fundamental sequence at the origin $\{U_{r_i}\}$ there exist some U_{r_j} and U_{r_k} belonging to $\{U_{r_i}\}$ such that $(g+U_{r_j}) \cap (f+U_{r_k}) = \phi$.

Lemma 1. *Let E be a T_1 pre-linear ranked space, all of whose neighbourhoods at the origin are symmetric ($U=-U$). Then the space E is a T_2-space.*