44. *On a Parametrix in Some Weak Sense of a First Order Linear Partial Differential Operator with Two Independent Variables*

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Introduction. Let $L=\partial/\partial t+i\phi(x)\sigma(t)\partial/\partial x$ be a first order linear partial differential operator with two independent variables in an open rectangle $\Omega=(a,b)\times(\alpha,\beta)\subset R_x^1\times R_t^1$, $-\infty\leqq a<b\leqq+\infty$, $-\infty\leqq\alpha<0<\beta\leqq+\infty$. In this paper we construct a parametrix of L in some weak sense and consider the regularity of the solution of the equation,

(0.1) $$Lu=f \quad \text{in } \Omega,$$

under the assumptions that

(0.2) $\phi\in C^\infty((a,b))$, and all derivatives of ϕ are bounded,

(0.3) $\sigma\in C^\infty((\alpha,\beta))$, $\sigma(t)\geqq 0$ in (α,β), and zeros of σ are all of finite order.

Equation (0.1) is locally solvable in Ω under these assumptions (cf. [1], [4]), but is not hypoelliptic in general (cf. [6]). In § 4 it will be seen how the regularity, with respect to t, of the solution u of (0.1) increases.

§ 1. Outline of the construction of a parametrix. We consider the solution of the form

(1.1) $$u(x,t)=\frac{1}{2\pi i}\int\exp\left(i\xi\int_0^t\sigma(s)ds\right)v(x,\xi)d\xi.$$

Calculating formally, we have

(1.2) $$Lu=\frac{\sigma(t)}{2\pi}\int\exp\left(i\xi\int_0^t\sigma(s)ds\right)(\xi v(x,\xi)+\phi(x)\partial/\partial x v(x,\xi))d\xi.$$

Remark that if $\sigma(t)>0$ in (α,β)

(1.3) $$g(t)=\frac{\sigma(t)}{2\pi}\int\exp\left(i\xi\int_0^t\sigma(s)ds\right)\left(\int\exp\left(-i\xi\int_0^{t'}\sigma(s)ds\right)g(t')dt'\right)d\xi$$

for every $g\in C_0^\infty((\alpha,\beta))$. Then, we can expect that when the solution v of the equation

(1.4) $$\xi v(x,\xi)+\phi(x)\partial/\partial x v(x,\xi)=\int\exp\left(-i\xi\int_0^{t'}\sigma(s)ds\right)f(x,t')dt'$$

is substituted in the right-hand side of (1.1) $u(x,t)$ will give a solution of (0.1).

§ 2. Preliminary lemmas. We state two lemmas for the construction of a parametrix of L without proof.

Lemma 2.1. *Let ϕ satisfy* (0.2). *We consider the equation*