61. On Micro-Analyticity of the Elementary Solutions of Hyperbolic Differential Equations with Real Analytic Coefficients

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In this note we state a theorem on (micro-) analyticity of the elementary solutions of hyperbolic differential equations with (not necessarily constant) multiple characteristics. Our result is a generalization of those of Kawai [1], Hörmander [2] and Andersson [3] which deal with operators with simple characteristics. (See Atiyah-Bott-Gårding [4] for operators with constant coefficients.)

If an m-th order differential operator $P(t, x, D_t, D_x)$ is hyperbolic with respect to the direction $(1, \cdots, 0)$, there exists a unique elementary solution of the Cauchy problem, that is, m-tuple of hyperfunctions $E_j(t, x)$ $(j=1, \cdots, m)$ such that

$$P(t, x, D_t, D_x)E_j(t, x)=0,$$
$$D_t^{i-1}E_j(0, x)=\delta_{ij}\delta(x) \qquad (i, j=1, \cdots, m).$$

(See Kawai [5] and Bony-Schapira [6].) Our problem is to decide the singular spectrum of $E_j(t, x)$.

Recently Kashiwara-Kawai [7] defined micro-hyperbolicity and constructed good elementary solutions for micro-hyperbolic operators. The essential key to our theorem is their deep analysis in micro-local sense. Remark that our lemma is valid for pseudo-differential operators.

Here we treat only the simplest case. More complete results and proofs will be published elsewhere.

First we set up a class of operators which can be easily handled. Let $P(x, D_x)$ be a pseudo-differential operator defined in a neighborhood of $x_0^*=(x_0, \xi_0) \in P^*X$. Let $\sigma(P)(x, \xi)=p_1^{s_1}(x, \xi) \cdots p_r^{s_r}(x, \xi)$ be an irreducible decomposition at x_0^*. We call $P(x, D_x)$ reductive if each $p_j(x, \xi)$ is simple characteristic, that is, $d_{(x,\xi)}p_j(x, \xi)$ is not parallel to $\sum_i \xi_i dx_i$. In this case we can define r-bicharacteristic strips through x_0^*. A hyperbolic differential operator is called reductive if it is reductive at each point on its real characteristic variety.

Examples.

$D_t^2-t^2(D_x^2+D_y^2)$

$(D_t^2-a(t, x, y)D_x^2-b(t, x, y)D_y^2)(D_t^2-c(t, x, y)D_x^2-d(t, x, y)D_y^2)$

where a, b, c and d is positive for real (t, x, y).