60. *Elements of Finite Order in an Ordered Semigroup Whose Product is of Infinite Order*

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We use the terminology and notation in [1] freely. By an *ordered semigroup* we mean a semigroup with a simple order which is compatible with the semigroup operation. Let a be an element of an ordered semigroup S. a is called *positive* [*negative*; *nonnegative*; *nonpositive*] if $a<a^2$ [$a^2<a$; $a\leq a^2$; $a^2\leq a$]. The number of distinct powers of a is called the *order* of a. The semigroup S is called *nonnegatively ordered* if all elements of S are nonnegative.

In [8], we gave the property that the set of all elements of finite order of a nonnegatively ordered semigroup S forms a subsemigroup of S, if it is nonempty. This property does not hold in general in ordered semigroups not necessarily nonnegatively ordered. In fact, Kuroki [2] gave the ordered semigroup K consisting of elements

$$e<x<u_1<u_2<\cdots<r_1<r_2<\cdots$$
$$<g<h<s_1<s_2<\cdots<y<v_1<v_2<\cdots<f$$

with the multiplication table

	e	x	u_j	r_j	g	h	s_j	y	v_j	f
e	e	e	e	e	e	e	e	e	e	e
x	e	e	e	e	e	e	u_j	r_1	r_{j+1}	g
u_i	e	e	e	e	e	e	u_{i+j}	r_{i+1}	r_{i+j+1}	g
r_i	e	u_i	u_{i+j}	r_{i+j}	g	g	g	g	g	g
g	g	g	g	g	g	g	g	g	g	g
h	h	h	h	h	h	h	h	h	h	h
s_i	h	h	h	h	h	h	s_{i+j}	v_i	v_{i+j}	f
y	h	s_1	s_{j+1}	v_j	f	f	f	f	f	f
v_i	h	s_{i+1}	s_{i+j+1}	v_{i+j}	f	f	f	f	f	f
f	f	f	f	f	f	f	f	f	f	f

and the ordered semigroup K' arising from K by identifying the elements g and h, as examples of ordered semigroups in which the elements x and y are elements of finite order but the element $r_1=xy$ is an element of infinite order.

In this paper we consider conversely and prove the following

Theorem. *Let x and y be elements of finite order of an ordered semigroup S such that $x\leq y$, $xy\leq yx$ and xy is a positive element of in-*