99. *Fourier Transforms on the Cartan Motion Group*

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The purpose of the present paper is to characterize the images of some function spaces on the Cartan motion group by the Fourier transform.

1. Preliminaries. Let G_0 be a connected non-compact semisimple Lie group with finite centre and $\mathfrak{g}$ be its Lie algebra. We fix a maximal compact subgroup K of G_0. Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ be the Cartan decomposition of $\mathfrak{g}$, where $\mathfrak{k}$ is the subalgebra corresponding to K. Then K operates on $\mathfrak{p}$ via the adjoint representation. Let G be the semidirect product of $\mathfrak{p}$ and K. The group G is called the Cartan motion group.

Let $\hat{\mathfrak{p}}$ be the dual space of $\mathfrak{p}$. Then K operates also on $\hat{\mathfrak{p}}$ via the contragredient representation of Ad, $\langle k\cdot\xi, X\rangle=\langle\xi, Ad(k)^{-1}X\rangle$ ($k\in K$, $\xi\in\hat{\mathfrak{p}}$ and $X\in\mathfrak{p}$). For any $\xi\in\hat{\mathfrak{p}}$ we can associate an irreducible unitary representation of $\mathfrak{p}$ by $X\to e^{i\langle\xi,X\rangle}$. We also denote it by ξ. We denote by U^{ξ} the unitary representation of G induced by $\xi\in\hat{\mathfrak{p}}$. Since the Killing form B on $\mathfrak{g}$ is positive definite on $\mathfrak{p}$, we can identify $\hat{\mathfrak{p}}$ with $\mathfrak{p}$. We denote by ξ_X the corresponding element in $\hat{\mathfrak{p}}$ to $X\in\mathfrak{p}$.

Let dk be the normalized Haar measure on K. Let $\mathfrak{H}=L^2(K)$. We denote by $\mathbf{B}(\mathfrak{H})$ the Banach space of all bounded linear operators on $\mathfrak{H}$. In $\mathfrak{p}$ and $\hat{\mathfrak{p}}$ we can define K-invariant measures which are induced by B. We normalize these measures by multiplying $(2\pi)^{-n/2}$ ($n=\dim\mathfrak{p}$) and denote them by dX and $d\xi$, respectively. We normalize the Haar measure dg on G such as $dg=dXdk$. For any $f\in L^1(G)$ we put

$$T_f(\xi)=\int_G f(g)U_g^{\xi}dg.$$

Then T_f is a $\mathbf{B}(\mathfrak{H})$-valued function on $\hat{\mathfrak{p}}$. It is called the Fourier transform of f.

2. Plancherel formula. Let $\mathfrak{a}$ be a maximal abelian subalgebra of $\mathfrak{g}$ contained in $\mathfrak{p}$. Fixing a lexicographic order in the dual space of $\mathfrak{a}$, we denote by P_+ the set of all positive restricted roots of the pair $(\mathfrak{g},\mathfrak{a})$. Let $\mathfrak{a}^+$ be the positive Weyl chamber in $\mathfrak{a}$. Since the Killing form B is positive definite on $\mathfrak{a}$, B gives rise to an euclidean measure dH on $\mathfrak{a}$. Let M be the centralizer of $\mathfrak{a}$ in K. We denote by dk_M the K-invariant measure on K/M induced by $-B$. We put $vol(K/M)=\int_{K/M}dk_M$. Let