143. *Exact Solution of a Certain Semi-Linear System of Partial Differential Equations related to a Migrating Predation Problem*

By Hidenori HASIMOTO

Department of Physics, University of Tokyo

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1. Introduction. This paper is concerned with the solution of the initial value problem for the system of equations for $u_1(x,t)$ and $u_2(x,t)$:

$$L_i[u_i] \equiv \left(\frac{\partial}{\partial t} + c_i \frac{\partial}{\partial x}\right) u_i = \lambda_i u_1 u_2, \qquad (i=1,2) \tag{1.1}$$

with the bounded and measurable initial data

$$u_i(x,0) = u_i^0(x), \; |x| < \infty. \tag{1.2}$$

The system (1.1) is the simplest hyperbolic one describing the non-linear coupling (characterized by parameters λ_1 and λ_2) between two waves propagating along the x-axis with constant velocities c_1 and c_2 respectively. If we put $c_1 = \lambda_1 = 1$ and $c_2 = \lambda_2 = -1$ it is reduced to the system proposed by Yamaguti [1] in order to describe a time history of the distribution of predator $u_1(t,x)$ and prey $u_2(t,x)$ running on a straight line in the opposite directions. Yamaguti [1] and Yoshikawa and Yamaguti [2] have given extensive studies of this system and have derived many important asymptotic properties of solutions as $t \to \infty$ without solving the equations explicitly. As far as the author is aware no explicit solution of our problem is found in the literature, in spite of the fact that it is reducible to the form amenable to Moutard's theorem [3].

The aim of this paper is to give the explicit solution of our problem and its version by means of a transformation analogous to that used by Hopf [4] and Cole [5] in their derivation of the solution of the Burgers equation. Several illustrating examples substantiating Yamaguti and Yoshikawa's prediction are given.

2. General solution. The solution u_i of (1.1) is derivable from the function ϕ:

$$u_i = \lambda_j^{-1} L_j[\phi], \qquad (j \neq i) = 1 \text{ or } 2 \tag{2.1}$$

provided that ϕ satisfies the equation

$$L_1 L_2[\phi] = L_1[\phi] L_2[\phi]. \tag{2.2}$$

Here and hereafter the suffices i and j denote the pair 1 and 2 or 2 and 1.