142. A Comment on the Galois Theory for Finite Factors

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1. As the case of simple rings, it is proved by Nakamura and Takeda ([6]–[8] and [9]) that a Galois theory holds true for finite factors under some conditions.

Throughout this paper, denote by $\mathfrak{H}$ a separable Hilbert space, by $\mathcal{A}$ a von Neumann algebra acting standardly on $\mathfrak{H}$, by G a countable discrete group of outer $*$-automorphisms of $\mathcal{A}$ and by $\mathcal{B}$ the fixed algebra of $\mathcal{A}$ under G, that is,

$$\mathcal{B}=\{A \in \mathcal{A};\, g(A)=A \text{ for all } g \in G\}.$$

Let $\mathcal{A}$ be a II_1-factor and G an outer automorphism group of $\mathcal{A}$. Then $\mathcal{A}$ is called a *Galois extension* of $\mathcal{B}$ with the *Galois group* G if $\mathcal{B}$ satisfies the condition:

(1) *The commutant* $\mathcal{B}'$ *of* $\mathcal{B}$ *is a* II_1*-factor.*

The fundamental theorem ([7, Theorem 2]) of the Galois theory for finite factors is the following:

Theorem A. *If* $\mathcal{A}$ *is a Galois extension of* $\mathcal{B}$ *with the Galois group* G*, then the lattices of all subgroups of* G *and of all intermediate subfactors* $\mathcal{B}$ *to* $\mathcal{A}$ *are dually isomorphic by the usual Galois correspondence.*

Furthermore the condition (1) is equivalent to the following condition:

(2) G *is finite*

([7, Theorem 3]).

The Galois theory for general von Neumann algebras is discussed by Haga and Takeda ([4]) or Henle ([5]).

In this paper, we shall show the following theorem as a comment of the Galois theory for II_1-factors.

Theorem 1. *Assume that* $\mathcal{A}$ *be a* II_1*-factor and* G *a finite group. Then the crossed product* $G \otimes \mathcal{A}$ *of* $\mathcal{A}$ *by* G *is isomorphic to the tensor product* $\mathcal{B} \otimes \mathcal{L}(l^2(G))$ *of* $\mathcal{B}$ *and the algebra of all bounded linear operators on the Hilbert space* $l^2(G)$.

Recently, M. Choda in [1] introduced a notion of shift for automorphism groups. Relating to it, we shall characterize the shift for finite groups of automorphisms.

2. Now, we shall relate briefly as to the crossed product according to Haga and Takeda [4].