141. *Conformal Sewings of Slit Regions*

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In this paper we shall consider the Riemann surfaces of *infinite genus* obtained by conformal sewings of slit regions and state some properties induced from the distribution of the slits. This note is an announcement of the results and the details will be given in another paper together with related topics.

1. Let R be a plane region and β be the boundary of R. Take mutually disjoint slits $\{\gamma_n\}_{n=1}^{\infty}$ in R clustering nowhere in R. Denote $G=R-\bigcup_n \gamma_n$. Now we shall define the conformal sewings. (1) Around each γ_j, take a parametric disk U_j so that γ_j is represented as $\{z_j\,;\,|\mathrm{Re}\, z_j|\leqq 1, \mathrm{Im}\, z_j=0\}$ by the local parameter z_j, denote by γ_j^+ and γ_j^-, the upper edge and the lower one of γ_j respectively, (2) partition $\{\gamma_n\}$ into finite sections $(\gamma_i)_{i\in I_k}$ $(k=1,2,\cdots)$ and (3) for each arrangement of elements in every finite section $(\gamma_i)_{i\in I_k}$, say $(\gamma_{i_1},\gamma_{i_2},\cdots,\gamma_{i_{n(k)}})$, identify γ_j^- with γ_{j+1}^+ $(j=i_1,i_2,\cdots i_{n(k)-1})$ and $\gamma_{i_{n(k)}}^-$ with $\gamma_{i_1}^+$ $(k=1,2,\cdots)$. Then we obtain a Riemann surface $S(G)$ and call such an operation a *conformal sewing* of G.

Definition. We say that slit region G belongs to class O_1 (resp. O_2) if $S(G)\in O_G$ for any (resp. some) conformal sewing of G.

Then we have,

Proposition 1. *There exists a slit region with an infinite number of slits, which belongs to class* O_1.

Now we introduce two families F^1 and F^0 of curves in $G-\overline{G}_0$, where G_0 is a parametric disk in G. F^1 consists of all c such that c is a finite union of closed Jordan curves in $G-\overline{G}_0$ and separates β from ∂G_0. While, $c\in F^0$ iff c is a finite union of closed curves which are closed or join some γ_n with γ_m and c separates β from ∂G_0. We denote by $\lambda(F^1)$ (resp. $\lambda(F^0)$) the extremal length of F^1 (resp. F^0).

Definition. We say that G is of weak (resp. semiweak) type if $\lambda(F^1)=0$ (resp. $\lambda(F^0)=0$). And we say G is of parabolic type if G has no non-constant HB-functions vanishing along $\partial G=\bigcup_n \gamma_n$.

We can prove by extremal length methods the following inclusion relations:

$$\begin{tikzcd}
O_1 \arrow[dr, shift left] & & & & \\
 & O_2 \arrow[ul, shift left, "/" marking] \arrow[dl, shift left, "/" marking] \arrow[r] & \text{semiweak} \arrow[r, shift left] & \hat{G}\in O_G \arrow[l, shift left] \arrow[r, shift left] & \text{parabolic} \arrow[l, shift left, "/" marking] \\
\text{weak} \arrow[uu, "/" marking] \arrow[ur, shift left] & & & &
\end{tikzcd}$$