138. *On the Strict Union of Ranked Metric Spaces*

By Shizu NAKANISHI
University of Osaka Prefecture

(Comm. by Kinjirô KUNUGI, M. J. A., Oct. 12, 1974)

The main purpose of this paper is to give the definition of the "*union*" of ranked spaces and to show that

(i) properties relating to convergence in the strict union E of ranked metric spaces $E_\alpha(\alpha \in A)$ are reduced to those in E_α (Theorem 1),

(ii) if $E_\alpha(\alpha \in A)$ is directed under set theoretic inclusion, a completion of E results from the completion of each E_α (Theorem 2).

Throughout of this paper, a "*ranked space*" means a ranked space of indicator ω_0(ω_0 is the first nonfinite ordinal) defined as follows:

1. Definition of the ranked space (of indicator ω_0). Let us consider a non-empty set E in which each point p has a non-empty family consisting of subsets of E, denoted by $V(p), U(p), \cdots$ and called *preneighborhoods* (sometimes called neighborhoods) *of* p or *p-preneighborhoods*, such that

(A) $p \in V(p)$.

We denote the family of all p-preneighborhoods by $\mathcal{V}(p)$, and put $\mathcal{V}=\{V(p); V(p) \in \mathcal{V}(p), p \in E\}$. Moreover, we assume that for each $n \in N$ ($N=\{0,1,2,\cdots\}$), we have a family $\mathcal{V}_n$ of preneighborhoods, called *preneighborhoods of rank n*, which satisfies the following axiom (a):

(a) For each preneighborhood $V(p)$ of p and for each number $n \in N$, there exist an m and a $U(p)$ which satisfy at the same time $U(p) \subset V(p)$, $U(p) \in \mathcal{V}_m$ and $n \leq m$. A *ranked space* (*of indicator* ω_0) is a non-empty set E endowed with these families $\mathcal{V}, \mathcal{V}_n(n \in N)$, which is written $(E, \mathcal{V}, \mathcal{V}_n)$ (briefly, $(E, \mathcal{V})$ or E). p-preneighborhoods of rank n are written $V(p, n)$.

We recall a few definitions concerning ranked spaces. $\{V(p_i, n_i); i=0,1,2,\cdots\}$ (briefly, $\{V(p_i, n_i)\}$) is called *fundamental* if $V(p_0, n_0) \supset V(p_1, n_1) \supset \cdots$ and for every i, there is $i_0 \geq i$ such that $p_{2i_0}=p_{2i_0+1}$ and $n_{2i_0}<n_{2i_0+1}$. E is called *complete* if for every fundamental sequence $\{V(p_i, n_i)\}$, $\cap V(p_i, n_i) \neq \phi$. $\{p_i\}$ is said to *r-converge to* p if there is a fundamental sequence $\{V(p, n_i)\}$ such that $p_i \in V(p, n_i)$ for every i. In this case, we write $p \in \{r\text{-lim}\, p_n\}$.

2. Definition of the union of ranked spaces. Let $(E_\alpha, \mathcal{V}^\alpha, \mathcal{V}_n^\alpha)$ $(\alpha \in A)$ be a family of the ranked spaces and let $E=\cup E_\alpha$. For each $p \in E$, we consider p-preneighborhoods in every E_α such that $p \in E_\alpha$,