137. On Isolated Components of Elements in a Compactly Generated l-Semigroup

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Recently, Murata and Hsu [2], [3] have presented analogous results of [4] for elements of an l-semigroup with a compact generator system. In [1] by defining an isolated component, authors have done a continued work of [4] to investigate the ideals which can be represented as the intersection of a finite number of f-primary ideals. The purpose of this note is to generalize results in [1] to elements in a compactly generated l-semigroup with a compact generator system.

Let L be a cl-semigroup with the following conditions as same as in [2], [3]:

(α) If M is a φ-system with kernel M^*, and if for any element a of L, M meets $\Sigma(a)$, then M^* meets $\Sigma(a)$.

(β) For any φ-primary element q of L, $q:q=e$. Moreover, if for any φ-system M, $\Sigma(r(q))$ meets M, then $\Sigma(q)$ meets M.

Throughout this note, we shall denote $r(a)$ as the φ-radical of an element a of L. Other terms are as same as in [2], [3].

1. Isolated components. Definition 1.1. Let a be an element of L and M be a φ-system. The isolated component $a(M)$ of a determined by M will be defined as the supremum of all $\{a:m\}, m$ runs over M, when M is not empty. $a(M)$ is defined to be a, when M is empty.

As in [3], we have assumed that there is such element x for any $a \in L$ and any $u \in \Sigma$ with $\varphi(u)\varphi(x) \leqq a, x \in \Sigma$. Then there exists such element $a:m$ in L and it can be seen from (3.2) in [3] that $a \leqq a(M)$.

Lemma 1.2. *Let M^* be any kernel of a φ-system M. If $x \in \Sigma(a(M))$, there exists an element m^* of M^* such that $\varphi(m^*)\varphi(x)$ is less than a.*

Proof. Since $x \in \Sigma(a(M))$, we have $x \leqq a(M) = \sup\left\{\bigvee_{m \in M} N_m\right\}$, when M is not empty (if M is empty, it is trivial), where N_m is the set of the compact elements u's such that $\varphi(m)\varphi(u) \leqq a$, and $\bigvee$ denotes the set-theoretic union. Then we can find a finite number of elements x_i of $\bigvee_{m \in M} N_m$ such that $x \leqq \bigcup_{i=1}^{n} x_i$. Suppose that $x_i \in N_{m_i}$, then $\varphi(m_i)\varphi(x_i) \leqq a$, $x \leqq \bigcup_{i=1}^{n} x_i \leqq \bigcup_{i=1}^{n} \varphi(x_i)$, $\varphi(x) \leqq \bigcup_{i=1}^{n} \varphi(x_i)$. Moreover, we can find m_i^* of M^*

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