126. *On the Sum of Digits of Prime Numbers*

By Iekata SHIOKAWA

Department of the Foundations of Mathematical Sciences,
Tokyo University of Education, Tokyo

(Comm. by Kôsaku YOSIDA, M. J. A., Oct. 12, 1974)

Let $r>1$ be a fixed integer. Then any positive integer n can be expressed in the form

$$n=\sum_{i=1}^{k} a_i r^{k-i}=a_1a_2\cdots a_k, \tag{1}$$

where each a_i is one of $0,1,\cdots,r-1$ and

$$k=k(n)=\left[\frac{\log n}{\log r}\right]+1, \tag{2}$$

where $[z]$ is the integral part of z. We put

$$\alpha(n)=\sum_{i=1}^{k} a_i.$$

I. Katai [2] proved, *assuming the validity of density hypothesis for the Riemann zeta function, that*

$$\sum_{p\le k}\alpha(p)=\frac{r-1}{2}\,\frac{x}{\log r}+O\left(\frac{x}{(\log\log x)^{\frac{1}{8}}}\right) \tag{3}$$

hold, where in the sum p runs through the prime numbers.

In this paper we shall prove without any unsolved hypothesis the result (3) of Katai, even with an improved remainder term. Our method is to appeal to a simple combinatorial argument, and the deepest result on which we shall depend is the well-known prime number theorem in a rather weak form.

In what follows all the O-constants depend possibly on the given scale r.

Theorem. *We have*

$$\sum_{p\le x}\alpha(p)=\frac{r-1}{2}\,\frac{x}{\log r}+O\left(x\left(\frac{\log\log x}{\log x}\right)^{\frac{1}{2}}\right),$$

where in the sum p runs through the primes.

Proof. Let b be any fixed positive integer not greater than $r-1$. For any positive integer n, we denote by $F(b,n)$ the number of b's appearing in the r-adic representation (1) of n and set

$$D(b,n)=\left|F(b,n)-\frac{k(n)}{r}\right|.$$

Thus we have

$$\sum_{p\le x}F(b,p)=\frac{1}{r}\sum_{p\le x}k(p)+O\left(\sum_{p\le x}D(b,p)\right). \tag{4}$$