163. *Kummer Surfaces in Characteristic 2*

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§0. **Introduction.** Let A be an abelian surface (i.e. abelian variety of dim 2) defined over a field of characteristic p ($p=0$ or a prime number). Denoting by ι the inversion of A ($\iota(u)=-u, u\in A$), we consider the quotient surface A/ι, which has only isolated singularities corresponding to the points of order 2 of A. When $p\neq 2$, A/ι has 16 ordinary double points and by blowing up these points, we get a $K3$ surface (i.e. regular surface with a trivial canonical divisor), called the Kummer surface of A.

When $p=2$, the situation is a little different. The number of singular points of A/ι is smaller (4, 2 or 1), but they are more complicated singularities. In this note, we consider the case where $A=E\times E'$ is a product of elliptic curves, and instead of directly looking at the singularities of A/ι and their resolution, we study the non-singular elliptic surface (Kodaira-Néron model) of the fibration $A/\iota\to E/\iota=\boldsymbol{P}^1$, induced by the projection $A\to E$. We define the *Kummer surface* of A, $Km(A)$, to be this non-singular elliptic surface, birationally equivalent to A/ι. Rather unexpectedly, we have

Proposition 1. *Assume $p=2$ and let $A=E\times E'$. Then*

(i) *$Km(A)$ (and hence A/ι) is a rational surface, if E and E' are supersingular elliptic curves.*

(ii) *$Km(A)$ is a $K3$ surface in all other cases.*

Proposition 2. *The Picard number ρ of $Km(A)$ in the case* (ii) *is given as follows:*

$$\rho=\begin{cases}18 & \text{if } E\not\sim E',\\ 19 & \text{if } E\sim E',\ End(E)=\boldsymbol{Z},\\ 20 & \text{if } E\sim E',\ End(E)\neq \boldsymbol{Z}.\end{cases}$$

Here "$\sim$" indicates isogeny. Note in particular that the $K3$ surfaces $Km(A)$ in (ii) cannot be supersingular in the sense of M. Artin [1], nor unirational (cf. [9]). It will be interesting to study the singularities of A/ι and to obtain its non-singular model for any abelian surface (or variety) in characteristic 2. For example, we can ask: (i) Is A/ι rational if A has no point of exact order 2? (In this case, A/ι is unirational.) (ii) Is A/ι birationally equivalent to a $K3$ surface if A has at least one point of exact order 2? We shall consider these questions in some occasion.