162. The Semi-discretisation Method and Nonlinear Time-dependent Parabolic Variational Inequalities

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1. Introduction. Let H be a (real) Hilbert space and X be a reflexive Banach space such that $X\subset H$, X is dense in H and the natural injection from X into H is continuous. We denote by X^* the dual space of X. Identifying H with its dual, we have the relations: $X\subset H\subset X^*$. Throughout this paper, let $0<T<\infty$, $1<p<\infty$ and $1/p+1/p'=1$. Let $K=\{K(t); 0\leq t\leq T\}$ be a family of closed convex subsets of X, ψ be a function on $[0,T]\times X$ such that for each $t\in[0,T]$, $\psi(t;\cdot)$ is a lower semicontinuous convex function on X with values in $(-\infty,\infty]$, and j be a continuous function on $[0,T]\times X$ such that for each $t\in[0,T]$, $j(t;\cdot)$ is convex on X. Suppose further that j is bounded on each bounded subset of $[0,T]\times X$ and for each $v\in L^p(0,T;X)$, $t\to\psi(t;v(t))$ is measurable. Then, for given $f\in L^{p'}(0,T;X^*)$ and $u_0\in X$ we mean by $V[K,j,\psi,f,u_0]$ the following problem: Find $u\in L^p(0,T;X)$ together with $u^*\in L^{p'}(0,T;X^*)$ such that

(i) u is an H-valued continuous function on $[0,T]$ with $u(0)=u_0$;

(ii) $u(t)\in K(t)$ for a.a. (almost all) $t\in(0,T)$ and $\psi(\cdot;u(\cdot))\in L^1(0,T)$;

(iii) $u^*(t)\in\partial j(t;u(t))$ for a.a. $t\in(0,T)$, where $\partial j(t;\cdot)$ is the subdifferential of $j(t;\cdot)$;

(iv) $u'=(\mathrm{d}/\mathrm{d}t)u\in L^2(0,T;H)$;

(v)
$$\int_0^T (u'(t),u(t)-v(t))_H\mathrm{d}t+\int_0^T (u^*(t)-f(t),u(t)-v(t))_X\mathrm{d}t \leq\int_0^T \{\psi(t;v(t))-\psi(t;u(t))\}\mathrm{d}t$$

for all $v\in L^p(0,T;X)\cap L^2(0,T;H)$ such that $v(t)\in K(t)$ for a.a. $t\in(0,T)$ and $\psi(\cdot;v(\cdot))\in L^1(0,T)$, where $(\cdot,\cdot)_X$ and $(\cdot,\cdot)_H$ stand for the natural pairing between X^* and X and the inner product in H, respectively.

Remark. If we take $\psi(t;\cdot)+I_{K(t)}(\cdot)$ instead of $\psi(t;\cdot)$ we can formulate the above problem without using $K(t)$, where $I_{K(t)}$ is the indicator function of $K(t)$.

Many results on the existence, uniqueness and regularity of solutions of this kind of problems have been established by many authors (e.g., [1], [2], [4]-[6], [8]-[11]). Brézis [2] and Moreau [6] treated the case where $\psi(t;\cdot)$ is the indicator function of $K(t)$; in this case, the domain