188. *Singularities of the Riemann Functions of Hyperbolic Mixed Problems in a Quarter-Space*

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Introduction. Matsumura [4] studied singularities of Riemann functions of hyperbolic mixed problems in a quater-space and determined the location of reflected waves by means of "localization theorem". In general Riemann functions also have singularities corresponding to lateral waves and boundary waves (see, Duff [3], Deakin [2]). Lateral waves arise from the presence of branch points in reflection coefficients and boundary waves are caused by real zeros of Lopatinski determinant. In this note we give a localization theorem which determines explicitly the location of lateral waves. The localization theorem of the fundamental solutions for the hyperbolic operators with constant coefficients in the whole space was established by Atiyah, Bott and Gårding [1].

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1. Assumptions and Riemann functions. Let $\boldsymbol{R}^n$ denote the n-dimensional Euclidean space and Ξ^n its complex dual space and write $x'=(x_1, \cdots, x_{n-1})$, $x''=(x_2, \cdots, x_n)$ for the coordinate $x=(x_1, \cdots, x_n)$ in $\boldsymbol{R}^n$ and $\xi'=(\xi_1, \cdots, \xi_{n-1})$, $\xi''=(\xi_2, \cdots, \xi_n)$ for the dual coordinate $\xi=(\xi_1, \cdots, \xi_n)$. The variable x_1 will play the role of "time", the variables $x_2, \cdots, x_n$ will play the role of "space". We shall also denote by $\boldsymbol{R}^n_+$ the half-space $\{x=(x', x_n) \in \boldsymbol{R}^n ; x_n>0\}$. For differentiation we will use the symbol $D=i^{-1}(\partial/\partial x_1, \cdots, \partial/\partial x_n)$.

Let $P=P(\xi)$ be a hyperbolic polynomial of order m of n variables ξ with respect to $\vartheta=(1, 0, \cdots, 0) \in \operatorname{Re} \Xi^n$ in the sense of Gårding. We consider the mixed initial-boundary value problem for the hyperbolic operator $P(D)$ in a quater-space

$$(1) \qquad P(D)u(x)=f(x), \qquad x \in \boldsymbol{R}^n_+,\ x_1>0,$$

$$(2) \qquad (D_1^k u)(0, x'')=0, \qquad 0 \leq k \leq m-1,\ x_n>0,$$

$$(3) \qquad B_j(D)u(x)|_{x_n=0}=0, \qquad 1 \leq j \leq l,\ x_1>0.$$

Here the $B_j(D)$ are boundary operators with order m_j. The number l of boundary conditions will be determined later on. We assume that the hyperplane $x_n=0$ is non-characteristic for $P(D)$ and $B_j(D)$.

Let $\operatorname{Re} A$ be the real hypersurface $\{\xi \in \operatorname{Re} \Xi^n ; P^0(\xi)=0\}$, where $P^0(\xi)$