186. *On the Broadwell's Model for a Simple Discrete Velocity Gas*

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In this paper we discuss the question of the global existence of non-negative solutions satisfying the semilinear hyperbolic system of equations

$$u_t + u_x = \varepsilon(w^2 - uv) \tag{1–1}$$

$$v_t - v_x = \varepsilon(w^2 - uv), \qquad (t, x) \in (0, +\infty) \times R \tag{1–2}$$

$$w_t = -2\varepsilon(w^2 - uv) \tag{1–3}$$

with the non-negative initial data

$$\begin{aligned} u(0, x) &= u_0(x) \\ v(0, x) &= v_0(x), \qquad x \in R \\ w(0, x) &= w_0(x). \end{aligned} \tag{2}$$

This system was proposed by J. E. Broadwell as one of the simplest models of a dilute gas whose molecules move in the discrete state. In this model, u and v are the numbers of molecules per unit volume with the velocities $(1, 0, 0)$ and $(-1, 0, 0)$ respectively, w is that with the velocity $(0, \pm 1, 0)$ or $(0, 0, \pm 1)$ and the gas motion is considered as one dimensional in x and homogeneous in y and z. A set (u, v, w) interacts only through binary collision with other molecules. As the collision coefficient ε is found to be proportional to the mutual-collision cross section, it may be taken as sufficiently small. A more detailed physical description of this model can be found in [1] and [3]. We remark that this approach gives the approximate solution of the Boltzmann equation in the meaning of restricting the molecular velocities to a finite set.

The local existence and uniqueness of the smooth or C^1-solution for the Cauchy problem (1) and (2) can be obtained as a classical result (see [2]). From now on, we denote the problem (1) and (2) by (C. Pr.).

As for the system (1), there exist the following relations which play an essential role to obtain the global solution of (C. Pr.); the conservation of mass:

$$(u + v + w)_t + (u - v)_x = 0 \tag{3}$$

the conservation of momentum:

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