181. *Cohomology of Vector Fields on a Complex Manifold*

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§ 1. Let M be a complex manifold. Let $\mathcal{A}$ denote the space of smooth vector fields of type (1, 0) on M. $\mathcal{A}$ is regarded as a Lie algebra under the usual bracket operation. Recently it is shown that the Lie algebra structure of $\mathcal{A}$ uniquely determines the complex analytic structure of M (I. Amemiya [1]), and thus it would be interesting to calculate the cohomology of the Lie algebra $\mathcal{A}$ associated with various representations. In this note, we shall state some results concerning the cohomology of the Lie algebra $\mathcal{A}$. Details will appear elsewhere.

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§ 2. We recall here briefly the definition of the cohomology group of a Lie algebra $\mathfrak{g}$ associated with a $\mathfrak{g}$-module W. Let $C^p(\mathfrak{g}; W)$ denote the space of alternating p-forms on $\mathfrak{g}$ with values in the vector space W for $p>0$; we put $C^0(\mathfrak{g}; W)=W$ and $C^p(\mathfrak{g}; W)=0$ for $p<0$. The coboundary operator $d: C^p(\mathfrak{g}; W)\to C^{p+1}(\mathfrak{g}; W)$ is defined by the following formula:

$$(d\omega)(X_1, \cdots, X_{p+1})=\sum_{i=1}^{p+1}(-1)^{i-1}X_i\omega(X_1, \cdots, \hat{X}_i, \cdots, X_{p+1})$$
$$+\sum_{i<j}(-1)^{i+j}\omega([X_i, X_j], X_1, \cdots, \hat{X}_i, \cdots, \hat{X}_j, \cdots, X_{p+1})$$

$(X_1, \cdots, X_{p+1}\in\mathfrak{g}, \omega\in C^p(\mathfrak{g}; W))$. The p-th cohomology group of this cochain complex $C(\mathfrak{g}; W)=\oplus_p C^p(\mathfrak{g}; W)$ will be denoted by $H^p(\mathfrak{g}; W)$. If the $\mathfrak{g}$-module W has a ring structure such that $X(fg)=(Xf)g+f(Xg)$ $(X\in\mathfrak{g},\ f, g\in W)$, then the total cohomology $H^*(\mathfrak{g}; W)=\oplus_p H^p(\mathfrak{g}; W)$ has a graded ring structure. (For more details, see [3].)

§ 3. The Lie algebra $\mathcal{A}$ has a representation on the ring $\mathcal{F}$ of smooth functions on M when the vector fields are identified canonically with the derivations on the ring $\mathcal{F}$. We shall denote by $C^p_\Delta(\mathcal{A}; \mathcal{F})$ the subspace of $C^p_\partial(\mathcal{A}; \mathcal{F})$ consisting of the elements ω such that $\operatorname{supp}(\omega(X_1, \cdots, X_p))\subset\bigcap_{i=1}^p \operatorname{supp}(X_i)$ $(X_1, \cdots, X_p\in\mathcal{A})$. Furthermore we shall denote by $C^p_\partial(\mathcal{A}; \mathcal{F})$ the subspace of $C^p_\Delta(\mathcal{A}; \mathcal{F})$ consisting of the elements ω such that, if $f\in\mathcal{F}$ is anti-holomorphic on an open subset U of M, then $\omega(fX_1, X_2, \cdots, X_p)=f\omega(X_1, X_2, \cdots, X_p)$ on U for any X_1, $X_2, \cdots X_p\in\mathcal{A}$. If we put $C_\Delta(\mathcal{A}; \mathcal{F})=\oplus_p C^p_\Delta(\mathcal{A}; \mathcal{F})$, and $C_\partial(\mathcal{A}; \mathcal{F})$ $=\oplus_p C^p_\partial(\mathcal{A}; \mathcal{F})$, then $C_\Delta(\mathcal{A}; \mathcal{F})$ and $C_\partial(\mathcal{A}; \mathcal{F})$ form a subcomplex of