35. *On Stationary Point Sets of $(Z_2)^k$-Manifolds*

By Katsuhiro KOMIYA
Yamaguchi University

(Comm. by Kenjiro SHODA, M. J. A., March 12, 1975)

1. Definitions. In order to state the results we define some notions.

Let G be a finite group, and $\mathscr{F}$, $\mathscr{F}'$ be families of subgroups of G with $\mathscr{F}\supset\mathscr{F}'$. An $(\mathscr{F}, \mathscr{F}')$-free G-manifold is a pair (M, φ) consisting of a compact differentiable manifold M and a differentiable G-action $\varphi: G\times M\to M$ on M such that

(i) if $x\in M$, then the isotropy group $G_x\in\mathscr{F}$, and

(ii) if $x\in\partial M$, then $G_x\in\mathscr{F}'$.

We may define the unoriented bordism module $\mathfrak{N}_*(G;\mathscr{F},\mathscr{F}')$, over the unoriented cobordism ring $\mathfrak{N}_*$, which consists of bordism classes of $(\mathscr{F},\mathscr{F}')$-free G-manifolds (see Stong [2]). If $\mathscr{F}'$ is empty, we write $\mathfrak{N}_*(G;\mathscr{F})$ for this module.

Let F be the stationary point set of a G-mainfold (M,φ), and $F=\bigcup_i F_i$ be the decomposition by the connected components. Let $(D(\nu_i),\varphi_i)$ be the G-manifold consisting of the normal disc bundle $D(\nu_i)$ of F_i and the G-action φ_i induced by φ. We suppose that any connected component F_i satisfies

$$[D(\nu_i),\varphi_i]=[F_i][D(V_i),\Psi_i]$$

in $\mathfrak{N}_*(G;\mathscr{F}_A,\mathscr{F}_P)$ for some positive dimensional G-representation (V_i,ψ_i), where $\mathscr{F}_A$ (resp., $\mathscr{F}_P$) is the family of all subgroups (resp., all proper subgroups) of G and $D(V_i)$ is the unit disc of V_i. We say in this case that *F has a trivial normal bundle in the weak sense.* When we further suppose that $\dim F_i=\dim F_j$ implies $(V_i,\psi_i)\cong(V_j,\psi_j)$ as G-representations, we say that *F has a trivial normal bundle* (in the sense of Conner-Floyd [1; § 42]).

2. Statement of results. In this note we study the case in which G is $(Z_2)^k$, the direct product of k copies of the multiplicative cyclic group $Z_2=\{1,-1\}$. We obtain the following results:

Theorem 1. *If the stationary point set F of a closed $(Z_2)^k$-manifold (M,φ) has a trivial normal bundle, then we obtain*

(i) $[F]=0$ *in* $\mathfrak{N}_*$, *and*

(ii) $[M,\varphi]=0$ *in* $\mathfrak{N}_*((Z_2)^k;\mathscr{F}_A)$.

Corollary 2 (Conner-Floyd [1: (31.3)]). *The stationary point set F of a positive dimensional closed $(Z_2)^k$-manifold can not consist of one point.*