# 55. Note on Strongly Regular Rings and $P_1$-Rings

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Throughout, $R$ $(\neq 0)$ will represent a ring. $R$ is called a *reduced ring*, if $R$ contains no non-zero nilpotent elements. As is well-known, in a reduced ring every idempotent is central and the left annihilator $l(T)$ of an arbitrary subset $T$ of the ring coincides with the right one $r(T)$. Following [4], $R$ is said to be left *s-unital*, if $RI=I$ for every left ideal $I$ of $R$, or equivalently, if every principal left ideal $(a|$ of $R$ coincides with $Ra$. Needless to say, every regular ring is left $s$-unital. A left $R$-module $U$ is defined to be *p-injective*, if for any $(a|$ and any $R$-homomorphism $f: (a|\to U$ there exists an element $u \in U$ such that $f(x)=xu$ for all $x \in (a|$ (cf. [5]). If $R$ is a regular ring then every left $R$-module is $p$-injective. Conversely, if every $(a|$ is $p$-injective then $R$ is a regular ring. In fact, the identity map $i: (a|\to(a|$ is induced by the right multiplication of some idempotent contained in $(a|$. If $R$ is a *$P_1$-ring*, i.e., if $aR=aRa$ for any $a \in R$, then the set $N$ of nilpotent elements coincides with $l(R)$ (cf. [3]). Similarly, if $aR=a^2R$ for any $a \in R$ then $N=l(R)$. While, if $aR\subseteq Ra^2$ for any $a \in R$, then $N$ coincides with $l(R^2)$ (cf. [2]). As to other terminologies used here, we follow [1].

Now, the purpose of this note is to prove the following theorems.

**Theorem 1.** (a) *The following conditions are equivalent*:

(1) *$R$ is a strongly regular ring.*

(2) *$R$ is a reduced ring such that every $(a|$ is either $l(b)$ with some $b$ or $Re$ with some idempotent $e$.*

(3) *$R$ is a left s-unital, left duo ring such that every irreducible left $R$-module is p-injective.*

(4) *$R$ is a left duo ring such that every $(a|$ is p-injective.*

(5) *$R$ is a semi-prime $P_1$-ring.*

(6) *$R$ is a semi-prime ring such that $aR=a^2R$ for any $a \in R$.*

(7) *$R$ is a semi-prime ring such that $aR\subseteq Ra^2$ for any $a \in R$.*

(b) *The following conditions are equivalent*:

(1) *$R$ is a strongly regular ring with* 1.

(2) *$R$ is a reduced ring such that every $(a|$ is $l(b)$ with some $b$.*

(3) *$R$ is a left duo ring with* 1 *such that every irreducible left $R$-module is p-injective.*

(4) *$R$ is a $P_1$-ring with* 1.

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