51. *Boundary Behavior of Harmonic Measures*

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Introduction. In the recent study of harmonic functions on an open Riemann surface, it is known that every canonical potential, especially every harmonic measure assumes a constant value quasi-everywhere (or except for harmonic measure zero) on each component of Kuramochi boundary. Such a property for boundary behaviors has been investigated by Kusunoki and some others.

Continuously they have investigated whether these boundary behaviors would characterize those functions. For Riemann surfaces with a finite number of boundary components, these characterizations of harmonic measures are trivially established. M. Watanabe shows that for Riemann surfaces with countably many boundary components, it is also true under some additional conditions, and that there exist Riemann surfaces with uncountably many boundary components in which these boundary behaviors do not always characterize those functions.

In the present paper, we shall show first simple examples showing that such characterizations can not be expected for Riemann surfaces with countably many boundary components. We should note that there are some differences between the characterization by using harmonic measure and that by using Kuramochi capacity. Our Example 1 is concerned with the former and is a planar region. On the other hand, the Riemann surface in Example 2 concerned with the latter is of infinite genus. And next we shall show that the characterization of harmonic measures by using Kuramochi capacity is established for Riemann surfaces with finite genus and countably many boundary components.

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1. Let R be a Riemann surface and $HC=HC(R)$ be the set of harmonic functions on R such that i) each u belongs to class HD, i.e. u is harmonic and has a finite Dirichlet integral, ii) u takes a constant value almost everywhere (i.e. except a set of harmonic measure zero) on each Kerékjártó-Stoïlow's boundary component of R. We shall use the same terminologies and notations as in Ahlfors-Sario [1]. We write

$$HM=\{u \in HD\,;\, du \in \Gamma_{hm}\}.$$