87. *Difference Approximation of Evolution Equations and Generation of Nonlinear Semigroups*

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We consider the following nonlinear evolution equation

(DE) $\quad (d/dt)u(t) \in Au(t), \qquad 0<t<T,$

where A is a (multi-valued) quasi-dissipative operator. In this note, we construct the solution of the evolution equation (DE) by the method of difference approximation. In addition, we give a generation theorem of nonlinear semigroups through the difference approximation. We sketch here our results. The details will be treated in [6].

1. Preliminaries. Let X be a real Banach space. For the multi-valued operator A, we use the following notations:

$$D(A)=\{x \in X\,;\, Ax \neq \phi\}, \qquad R(A)=\bigcup_{x \in D(A)} \{y\,;\, y \in Ax\},$$

$$\text{and} \quad |||Ax|||=\inf\{\|y\|\,;\, y \in Ax\} \qquad \text{for } x \in D(A).$$

We identify the multi-valued operator A with its graph, so that we write $[x, y] \in A$ if $y \in Ax$.

Let F be the duality map from X into X^*. Then we set

$$\langle y, x\rangle_i=\inf\{\langle y, f\rangle\,;\, f \in F(x)\} \qquad \text{for } x, y \in X.$$

Let $A \subset X \times X$. A is said to be *dissipative* if for any $[x_i, y_i] \in A$ $(i=1,2)$,

$$\langle y_1-y_2, x_1-x_2\rangle_i \leq 0.$$

According to Takahashi [9], we introduce the following notion as a generalization of that of dissipative operators.

Definition 1. Let $A \subset X \times X$. A is said to be *quasi-dissipative* if for any $[x_i, y_i] \in A$ $(i=1,2)$,

$$\langle y_1, x_1-x_2\rangle_i+\langle y_2, x_2-x_1\rangle_i \leq 0.$$

The following example shows that quasi-dissipative operators are not always dissipative.

Example (I. Miyadera). Let $X=R^2$ with the maximum norm. Let $x_1=(1,1)$ and $x_2=(0,0)$. We set $D(A)=\{x_1, x_2\}$, $Ax_1=\{(\alpha, \beta)\,;\, \alpha \leq 0$ or $\beta \leq 0\}$ and $Ax_2=\{(\alpha, \beta)\,;\, \alpha \geq 0$ or $\beta \geq 0\}$. Then A is quasi-dissipative in X but $A-\omega$ is not dissipative in X for any real ω. In addition, $R(I-\lambda A) \supset D(A)$ for any $\lambda>0$.

The following plays a central role in our argument.

Lemma 1. *Let $A \subset X \times X$. Then the following are equivalent:*

(i) *A is quasi-dissipative;*