116. On Extensions of my Previous Paper "On the Korteweg-de Vries Equation"

By Masayoshi TSUTSUMI
Department of Applied Physics, Waseda University

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1. Introduction. Previously, in [1] we have proved the following result: *Let* $\{\varphi_j(x;t)\}$ *and* $\{\lambda_j(t)\}$, $j=1,2,\cdots$, *be a complete system of normalized eigenfunctions and eigenvalues, respectively, of the Schrödinger eigenvalue problem in* T^1, T^1 *being a torus, with* t *considered as a parameter*:

$$(1.1)\qquad \begin{cases} \dfrac{d^2}{dx^2}\varphi_j(x;t)+u(x,t)\varphi_j(x;t)=-\lambda_j(t)\varphi_j(x;t), \\ \varphi_j(\cdot,t)\in C^2(T^1), \qquad \text{for } \forall t\in(-\infty,\infty), \end{cases}$$

where $u(x,t)$ *is a real function belonging to* $C^\infty(T^1\times R^1)$. *Then we have the asymptotic expansion*:

$$(1.2)\qquad \sum_{j=1}^{\infty} e^{-\lambda_j(t)s}(\varphi_j(x,t))^2 \sim \sum_{i=0}^{\infty} s^{i-1/2}P_i(u,\partial u/\partial u,\cdots,\partial^{2(i-1)}u/\partial x^{2(i-1)})$$

where P_i *are uniquely determined and can be calculated explicitly in terms of the function* u *and its partial derivatives in* x, *of order* $\leqq 2(i-1)$. *If* $u=u(x,t)$ *evolves according to the equation*

$$(1.3)\qquad \begin{cases} \dfrac{\partial u}{\partial t}=\displaystyle\sum_{i=1}^{M} f_i(t)\frac{\partial}{\partial x}P_i(u,\cdots,\partial^{2(i-1)}u/\partial x^{2(i-1)}), \\ u(x,t)\in C^\infty(T^1\times R^1), \end{cases}$$

where M *is an arbitrary fixed positive integer and* $f_i(t)$ *are arbitrary smooth function of* t, *then the eigenvalues* $\lambda_j(t)$ *of the associated eigenvalue problem* (1.1) *are constants in* t *and every* $P_i(\cdot)$ *appeared in* (1.2) *is the conserved density of* (1.3).

In this note, two extensions of the above result are considered. One is to extend it into $n\times n$ matrix form. The other is to extend it into the case of many space variables.

2. $n\times n$ matrix form. Let $U(x,t)$ be a $n\times n$ Hermitian matrix function whose elements belong to $C^\infty(T^1\times R^1)$. Below, we denote the set of such matrix functions by $\boldsymbol{C}^\infty(T^1\times R^1)$. Consider the eigenvalue problem for the following matrix differential equation with t considered as a parameter:

$$(2.1)\qquad \begin{cases} \dfrac{d^2}{dx^2}\Phi+U(x,t)\Phi=-\lambda\Phi, \qquad -\infty<x,t<+\infty, \\ \Phi(\cdot;t)\in \boldsymbol{C}^2(T^1) \qquad \text{for all } t\in(-\infty,\infty). \end{cases}$$