108. *On the C^∞-Goursat Problem for 2nd Order Equations with Real Constant Coefficients*

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§ 1. **Introduction.** We consider the following Goursat problem (1.1)–(1.2).

$$\partial_t \partial_x u = \sum_{\substack{i+j+|\alpha|\leqslant 2 \\ i+j\leqslant 1}} a_{ij\alpha} \partial_t^i \partial_x^j \partial_y^\alpha u, \qquad t \in R_+^1,\ x \in R^1,\ y \in R^n \tag{1.1}$$

where $a_{ij\alpha}$ are real constants

$$\begin{cases} u(0, x, y) = \varphi(x, y) \in \mathcal{E}_{xy} \\ u(t, 0, y) = \psi(t, y) \in \mathcal{E}_{ty} \qquad t \geqslant 0 \\ \varphi(0, y) = \psi(0, y) \qquad \text{(compatibility condition).} \end{cases} \tag{1.2}$$

We notice that, $t=0$ and $x=0$ are characteristic hypersurfaces of the equation (1.1). We say that the Goursat problem (1.1)–(1.2) is well posed for the future in the space $\mathcal{E}$, if for any given Goursat data (1.2), there exists a unique solution $u(t, x, y) \in \mathcal{E}_{txy}$, $t \geqslant 0$, which takes the given Goursat data at $t=0$ and $x=0$.*)

Let us consider the characteristic equation (considering the lower order terms) of (1.1).

$$\lambda \xi = \sum_{\substack{1 \leqslant i+j+|\alpha| \leqslant 2 \\ i+j \leqslant 1}} a_{ij\alpha} \lambda^i \xi^j \eta^\alpha, \qquad \xi \in R^1,\ \eta \in R^n.$$

Then we have

$$\lambda = \sum_{j \leqslant 1,\ 1 \leqslant j+|\alpha| \leqslant 2} a_{0j\alpha} \xi^j \eta^\alpha \Big/ \Big(\xi - \sum_{|\alpha| \leqslant 1} a_{10\alpha} \eta^\alpha\Big). \tag{1.3}$$

Our purpose is to prove the following

Theorem 1. *The necessary and sufficient condition for the $\mathcal{E}$-wellposedness of the Goursat problem* (1.1)–(1.2) *in the neighborhood of the origin is that λ in* (1.3) *remains bounded when $|\xi|+|\eta|$ remains bounded.*

Remark 1. We can rewrite (1.1) in the following.

$$\{\partial_t - (a_1 \partial_{y_1} + a_2 \partial_{y_2} + \cdots + a_n \partial_{y_n} + a_0)\}\{\partial_x - (b_1 \partial_{y_1} + \cdots + b_n \partial_{y_n} + b_0)\} u = \sum_{|\alpha| \leqslant 2} c_\alpha \partial_y^\alpha u. \tag{1.4}$$

The necessary and sufficient condition in the theorem 1 is equivalent to $c_\alpha = 0$ for $|\alpha| \geqslant 1$.

§ 2. **Proof of Theorem 1.** At first we consider the following fairly simple equation;

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*) According to Banach's closed graph theorem, if the Goursat problem is $\mathcal{E}$-wellposed then the linear mapping $(\varphi, \psi) \to u$ is continuous from $\mathcal{E}_{xy} \times \mathcal{E}_{ty}$ into $\mathcal{E}_{txy}$.